Exam Code: 112416 **Paper Code: 6175**
(30)

Programme: Bachelor of Science (Honours) Mathematics
Semester-VI

Course Title: Complex Analysis

Course Code: BOML-6331

Time Allowed: 3 Hours **Max Marks: 80**

Note: Attempt five questions in all, selecting at least one question from each section. The fifth question may be attempted from any section. Each question carries equal marks.

Section-A

1(a) Show that the function $\sqrt{|xy|}$ is not analytic at origin, although the Cauchy- Riemann equations are satisfied at that point.

(b) Obtain Cauchy –Riemann Equations for an analytic function in Cartesian form. (8,8)

2(a) Show that the function $|z|^2$ is continuous everywhere but differentiable at origin only.

(b) If $u - v = (x - y)(x^2 + 4xy + y^2) and\ f(z) = u + iv$ is an analytic function of $z = x + iy, find\ f(z)$ in terms of z. (8,8)

Section-B

3(a) State and Prove Cauchy Integral formula in multi connected region.

(b) Evaluate $\int_C \frac{e^{2z}}{(z+1)^4} dz$ where C is the circle |z|= 3 (8,8)

4(a) If a function $f(z)$ is analytic for all finite values of z and is bounded then prove that $f(z)$ is constant.

(b) State and prove Morera's theorem. (8,8)

Section-C

5 State and prove Laurent's theorem and Obtain Laurent's series for $\frac{1}{z(z^2-3z+2)}$ in $1 < |z| < 2$ (16)

6(a) Find the nature of singularities and residues of the function $f(z) = \frac{z}{1+z^4}$

(b) If a function $f(z)$ is analytic except at finite number of singularities (including at infinity), then prove that the sum of the residues of these singularities is zero. (8,8)

Section-D

7(a) Use Rouche's theorem to determine the number of roots of the equation $z^8 - 4z^5 + z^2 - 1 = 0$

(b) Use Rouche's theorem, prove that every polynomial of degree n has exactly n zeroes. (8,8)

8(a) Find the general bilinear transformation which transform the unit circular disc. $|z| \leq 1$ on to $|w| \leq 1$

(b) Prove that every bilinear transformation maps circle or straight line into a circle or straight line.

(8,8)

Exam Code: 112416 **Paper Code: 6176**

(30)

Programme: Bachelor of Science (Honours) Mathematics Semester-VI

Course Title: Analytical Skills

Course Code: BOML-6332

Time Allowed: 3 Hours **Max Marks: 80**

Note: Attempt five questions in all, selecting at least one question from each section. Fifth question can be attempted from any section. Each section carries equal marks. (16 marks each)

Section-A

Q.1 (a) Find the wrong number from the series : 1,3,10,21,64,129,356,777

(b) Find the missing no. from the series : 165,195,255,285,345,______

(c) In this question there is some relationship between the two terms to the left of :: and the same relationship holds between the two terms to its right . Find out this term

NOPQ: MLKJ ::HIJK: ?

(a) DEFG (b) EFGH (c) FEDC (d) GFED

(d) In this question there is some relationship between the two terms to the left of :: and the same relationship holds between the two terms to its right . Find out this term

583: 293:: 488: ? **(4,4,4,4)**

Q.2 (a) What was the day of week on 28^{th} may, 2006?

(b) At what time between 4 and 5 o'clock will the hands of a clock be at the right angles?

(c) Following question is based on the information given below :

P×Q *means* P is the father of Q, P-Q *means* P is the sister of Q , "

$P + Q$ means P is the mother of Q , P ÷Q" *means* "*P is the brother of Q*"

Then what " J is the son of F "represent ? **(5,6,5)**

Section-B

Q.3 (a) Find the largest number which when subtracted from 10000, the remainder is divisible by 32,36,48 and 50 .

(b) Find the value of $[\,35.7 - \left(3 + \frac{1}{3+\frac{1}{3}}\right) - \left(2 + \frac{1}{2+\frac{1}{2}}\right)]$

(c) From a group of boys and girls , 15 girls leave. There are then left 2 boys for each girl . After this , 45 boys leave . There are then 5 girls for each boy. Find the number of girls in the beginning. **(6,5,5)**

Q.4 (a) The dimension of a rectangular room when increased by 4 metres, are in the ratio of 4:3 and when decreased by 4 metres , are in the ratio of 2:1 . Find the dimensions of the room.

(b) 2 men and 3 boys can do a piece of work in 10 days while 3 men and 2 boys can do the same work in 8 days . In how many days can 2 men and 1 boy do the work ?

(c) By walking at $\frac{3}{4}$ of his usual speed , a man reaches his office 20 minutes later than his usual time. Find the usual time taken by him to reach his office. (5,5,6)

Section-C

Q.5 (a) A circular wire of diameter 42cm is bent in the form of a rectanglewhose4 sides are in ratio 6:5 . Find the area of the rectangle .

(b) Find the volume , Curved surface area and total surface area of a cylinder with diameter of the base 7cm and height 40cm.

(c) How many spherical bullets can be made out of a lead cylinder 28cm high and with base radius 6cm, each bullet being 1.5cm in diameter? (5,5,6)

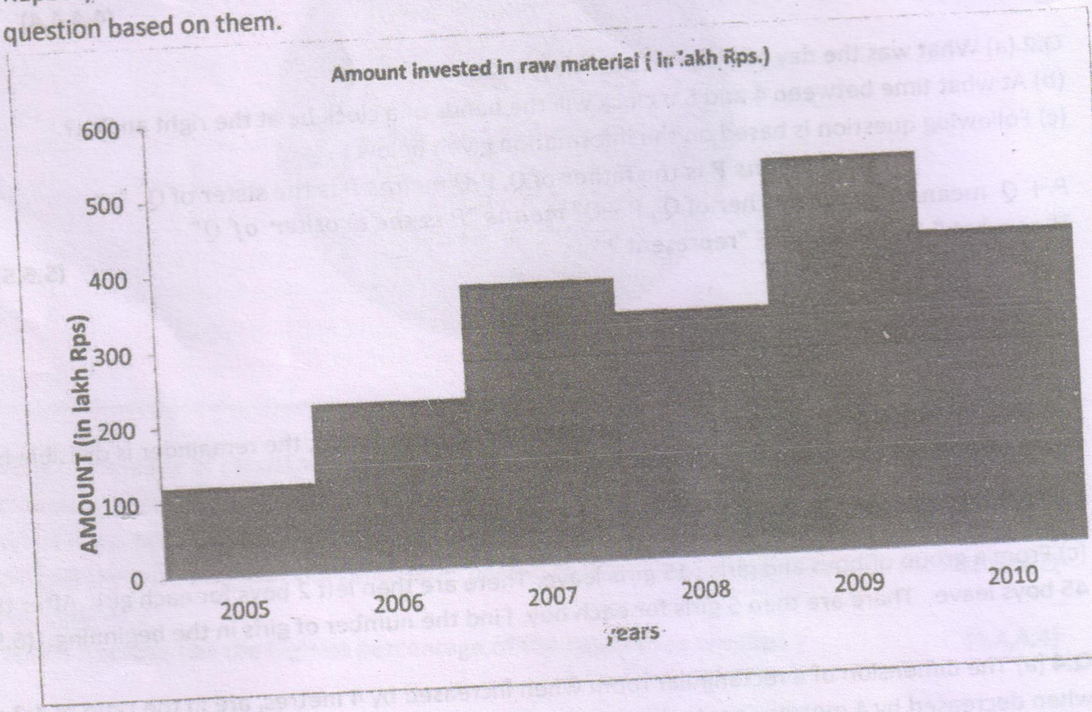
Q.6 (a) What annual instalment will discharge a debt of Rps. 1092 due in 3 years at 12% simple interest ?

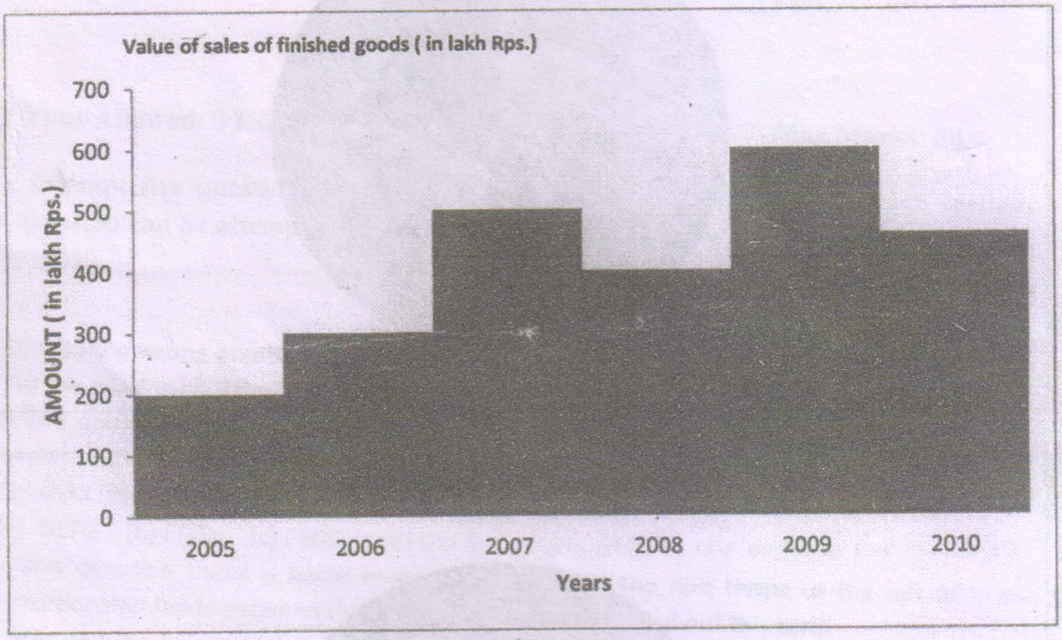
(b) A invested Rupee 76000 in a business . After few months , B joined him with Rupe 57000. At the end of the year , the total profit was divided between them in the ratio 2:1 . After how many months did B join?

(c) If $z = \frac{x^2}{y}$ and x,y are both increased in value by 10% , find the percentage change in the value of z (6,5,5)

Section-D

Q.7 (a) Out of the bar graph given below, one shows the amount (in lakhs Rps.) invested by a company in purchasing raw materials over the years and the other shows the value (in lakhs Rupees)of finished goods sold by the company over the years . Study the bar graph and answer the question based on them.





(a) In which year, there is maximum percentage increase in the amount invested in raw material as compared to the previous years ?

(b) In which year, the percentage change (compared to previous years) in the investment on raw materials is the same as that in the value of sales of finished goods?

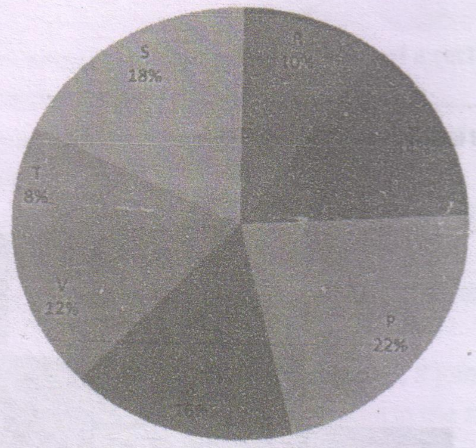
(c) What was the difference between the average amount invested in raw material during the given period and the average value of the sales of finished goods during this period?

(d) The value of sales of finished goods in 2009 was approximately what percent of the average amount invested in raw materials in the years 2007, 2008 and 2009? **(4,4,4,4)**

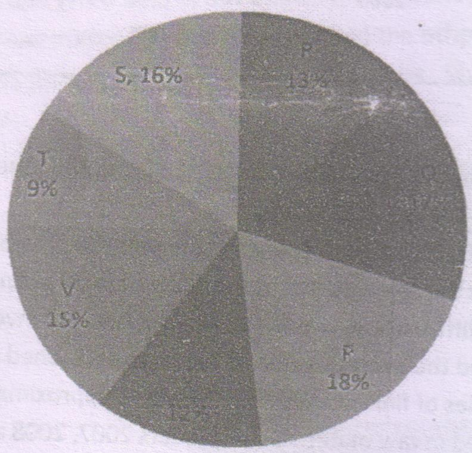
Q.8 Study the following pie graph carefully and answer the question given below

Distribution of the candidates enrolled for M.B.A entrance examination and those who passed in different institutes : P,Q,R,S,T,V,and X

CANDIDATES ENROLLED : 8550



CANDIDATES WHO PASSED : 5700



(a)What percentage of candidates passed the exam from the institute T out of the total number of candidates enrolled from the same institute ?

(b) What is the ratio of the candidates passed to the candidates enrolled from institute P?

(c) What is the percentage of the candidates passed to the candidates enrolled for the institute Q and R together ?

(d) Which institute has the highest percentage of the candidates enrolled ? **(4,4,4,4)**

Exam Code: 112416 **Paper Code: 6177**
(30)

Programme: Bachelor of Science (Honours) Mathematics
Semester-VI
Course Title: Numerical Analysis
Course Code: BOML-6333

Time Allowed: 3 Hours **Max Marks: 80**

Note: - Attempt five questions in all, selecting at least one question from each section. The fifth question may be attempted from any section. All question carries equal(8 marks each) marks. The students can use only Non programmable & Non Storage type calculator.

SECTION-A

Q1. (a) Find the negative real root of equation $x^2 - 4x - 10 = 0$ correct to one decimal places, using bisection method.

(b) The numbers 1.245, 102.179, 63.6, 22.00 are correct to last digit. Find the product. [8 X 2=16]

Q2. (a) Find the order of convergence of Newton Raphson Method.

(b) Find the double root of the equation $f(x) = x^4 - 11x^3 + 36x^2 - 16x - 64 = 0$ correct to two decimal places which is near 3.9.

[8 X 2=16]

SECTION-B

Q3. (a) Solve the equations x +2y + z = 8, 2x + 3y + 4z = 20, 4x +3y +2z =16 by Gauss Jordan method.

(b) Solve the equations by Gauss Seidel method (use initial approximation as (0, 0, 0))
5x + 2y + z = 12, x + 4y + 2z = 15, x+2y + 5z = 20. [8 X 2=16]

Q4. (a) Using Newton's forward difference formula, find the sum of $S_n = 1^3 + 2^3 + 3^3 + \cdots + n^3$.

(b) Use Newton's divided difference formula to find the value of y when x=2 for (1, -3), (3, 9), (4, 30) and (6, 132). [8 X 2=16]

SECTION-C

Q5 (a) Find the first and second order derivatives of $y = f(x) at\ x = 5$ for the given data.

x	0	2	3	4	7	9
f(x)	4	26	58	112	466	922

(b)) From the following table estimate the number of persons earning wages between Rs 60 and 70:

[8 X 2=16]

Marks	Below 40	40-60	60-80	80-100	100-120
No. of students	18	40	64	50	28

Q6 (a) Evaluate integral $I = \int_0^1 (1+x)^{-1}\ dx$ by using Gauss Legendre three-point formula.

(b) Evaluate $I = \int_0^1 \frac{1}{x+y} dx$ using Romberg's method correct to three decimal places. [8 X 2=16]

SECTION-D

Q7 (a) Using Picard's method, find an approximate value of y when $x = 0.4$, given that

$\frac{dy}{dx} = x^2 + y^2$ *and* $y = 0$ *when* $x = 0$ (*take* $h = 0.4$). [8 X 2=16]

(b) Tabulate the solution of $\frac{dy}{dx} = x + y$ with y (0)=0, for $0.4 < x \leq 1$ *with* $h = 0.1$, by Predictor Corrector Method.

Q8. (a) Apply Runge- Kutta method of order 4 to find approximate value of y for x = 0.4, in steps of 0.2, if

$\frac{dy}{dx} = x^2 + y^2$ *and* $y = 0$ *when* $x = 0$.

(b) Using Taylor series method, find an approximate value of y when $x = 0.1$, given that

$\frac{dy}{dx} = x - y^2$ *and* $y = 1$ *when* $x = 0$ (*take* $h = 0.1$). [8 X 2=16]

Exam Code: 112416 Paper/Subject Code: ~~6199~~ 6178

(40)

Programme: BACHELOR OF SCIENCE (HONOURS) MATHEMATICS SEMESTER-VI

Course Title: Special Functions

Course Code: BOML-6334

Time allowed: 3 hours **Maximum marks: 80**

Note: Candidates are required to attempt five questions, selecting at least one question from each section. The fifth question may be attempted from any section. All questions carry equal marks.

Section-A

Q1 (a) State and Prove Kummer's Theorem **10**

(b) Find the solution of $x(1-x)y''+\left(\frac{3}{2}-2x\right)y'+2y=0$ about $x=0$. **6**

Q2 (a) Show that $\frac{2}{\pi}\int_0^{\frac{\pi}{2}}\left(1-x^2\sin^2\theta\right)^{\frac{1}{2}}d\theta = {}_2F_1\left(-\frac{1}{2},\frac{1}{2};1;x^2\right), |x|<1$ **8**

(b) Show that $F(a;b;c;z)=\frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)}\int_0^1(1-t)^{c-b-1}t^{b-1}(1-zt)^{-a}dt$ **8**

Section-B

Q3 (a) State and prove generating function of Bessel's function **10**

(b) Prove that $\exp\left(\frac{1}{2}x\left(z-\frac{1}{z}\right)\right)=\sum_{n=-\infty}^{\infty}z^nJ_n(x)$ **6**

Q4 (a) Show that when n is positive integer $J_{-n}(x)=(-1)^nJ_n(x)$ **8**

(b) Show that $J_n(x)=\frac{1}{\pi}\int_0^{\pi}\cos(n\phi-x\sin\phi)d\phi$, where n is a positive integer. **8**

Section-C

Q5 (a) Express $x^4 + 2x^3 + 2x^2 - x - 3$ in terms of Legendre polynomials. 6

(b) State and Prove generating function for Legendre polynomials 10

Q6 (a) Prove that $P_n(0) = 0$, if n is odd. 8

(b) Show that $n P_n = (2n-1)x P_{n-1} - (n-1)P_{n-2}$, $n \geq 2$. 8

Section-D

Q7 (a) State and prove Rodrigues formula for $H_n(x)$ 10

(b) Prove that $H_n(x) = 2^n \exp\left(-\frac{1}{4}\frac{d^n}{dx^n}\right)x^n$ 6

Q8 (a) Prove that $H_n'(x) = 2n H_{n-1}(x)(n \geq 1); H_0'(x) = 0$ 8

(b) Show that Hermite polynomial are orthogonal over $(-\infty, \infty)$ w.r.t. the weight function e^{-x^2}. 8

Exam Code: 112416 **Paper Code: 6179**

(30)

Programme: Bachelor of Science (Honours) Mathematics
Semester-VI

Course Title: Differential Geometry

Course Code: BOML-6335

Time Allowed: 3 Hours **Max Marks: 80**

There are total Eight questions of equal marks (16 marks), two in each of the four Sections (A-D). Candidates are required to attempt five questions, selecting at least one question from each section. The fifth question may be attempted from any section.

Section A

1. (a) Write a note on three fundamental unit vectors associated with a space curve. Also discuss their relation with three fundamental planes. 8

(b) Find the equation of tangent line to a curve at a given point in vector and Cartesian form. 8

2. (a) Prove that the tangent at any point of the curve whose equations are $x = 3t, y = 3t^2, z = 2t^3$ makes a constant angle with the line $y = z - x = 0$. 8

(b)Show that the curve $x = au + b, y = cu + d, z = u^2$ has the same osculating plane at all the points. 8

Section B

3. (a) The necessary and sufficient condition that a given curve to be a straight line is that curvature is zero at all the points of the curve. 8

(b) Show that the curve $\vec{r} = \left(t, \frac{1+t}{t}, \frac{1-t^2}{t}\right)$ lies in a plane 8

4. (a) If the curvature κ of the Curve C is constant, then the curvature of the locus of the centre of the sphere of curvature is also constant. 8

(b) Find the curvature and torsion of the spherical indicatrix of the tangents. 8

Section C

5. (a) Define two parameter family of surfaces and envelopes. Also discuss the working rule to find the envelopes for two parameter family. 8

(b) Find the fundamental magnitudes for the Monge's form of the surface $z = f(x, y)$ 8

6. Find the equation of the tangent plane to the surface $f(x, y, z) = 0$. Using this find the tangent plane to the surface $z = x^2 + y^2$ at the point $(1, -1, 2)$. 16

Section D

7. (a)There exists no distinction between contravariant and co-variant vectors when we restrict ourselves to rectangular Cartesian transformation of coordinates. 8

(b) The sum and difference of two tensors of same kind result in a third tensor of same kind . 8

8 (a) Write a detailed note on first fundamental and second fundamental tensors. 8

(b) State and prove quotient law of tensors. 8