COE - OFFICE
KMV - II [N.S.B]
MOR - 8/5/2024

Exam Code: 112414
(30)

Paper Code: 4205

Programme: Bachelor of Science (Honours) Mathematics
Semester-IV

Course Title: Vector Calculus

Course Code: BOML-4331

Time Allowed: 3 Hours **Max Marks: 80**

Note: Attempt five questions, selecting at least one question from each section. The fifth question may be attempted from any section. Each question carry equal marks.(16 marks each)

Section A

Q1. (a) If a, b, c *and* d are four vectors, show that $[(a \times b) \times (a \times c)].d = (a.d)[a, b, c]$.

(b) Prove that the derivative of a vector of constant magnitude is perpendicular to the vector.

[8 X 2=16]

Q2. (a) Calculate the angle between the surfaces $x^2 + y^2 + z^2 = 9$ *and* $x^2 + y^2 - 3 = z$ at the point $(2, -1, 2)$.

(b) Find the maximum value of directional derivative of $f = x^2z + xy^2 + yz^2$ in the direction of vector *at the point* $(1, 3, 2)$. Also find the direction in which it occurs.

[8 X 2=16]

Section B

Q3. (a) Is $\nabla^2\left(\frac{1}{r}\right) = 0$, where the symbols have the usual meaning. Justify your answer.

(b) Check whether $curl\ curl\ \vec{v} = grad\ div\ \vec{v} - \nabla^2\vec{v}$ *or not.*

[8X2=16]

Q4. (a) Show that divergence of $\vec{g} \times \vec{r} = 0$ *if* $\nabla \times \vec{g} = \vec{0}$.

(b) Check whether the vector $3y^2z^2\,\hat{\imath} + 4x^2z^2\hat{\jmath} + 3x^2y^2\,\hat{k}$ is solenoidal or not.

[8X2=16]

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Section C

Q5. (a) If $\vec{g} = 2y\,\hat{\imath} - 2\hat{\jmath} + x\hat{k}$, evaluate $\oint_C \vec{g} \times \overrightarrow{dr}$, along the curve $x = \cos\theta, y = \sin\theta, z = 2\cos\theta\ from\ \theta = 0\ to\ \frac{\pi}{2}$.

(b) Express Laplacian operator of any $\vec{A}$ in orthogonal coordinates. [8 X 2=16]

Q6. (a) Verify Green's theorem in plane for $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$, around the boundary C of the region enclosed by $x^2 = y\ and\ \sqrt{x} = y$.

(b) Find the circulation of $\vec{f} = y\,\hat{\imath} + z\,\hat{\jmath} + x\,\hat{k}$ along the circle in the xy plane $x^2 + y^2 = 1$, $z = 0$.
[8 X 2=16]

Section D

Q7. Verify Stoke's Theorem for a vector field defined by $\vec{F} = (xy)\,\hat{\imath} + xy^2\,\hat{\jmath}$, over the square with vertices $(1,0)$, $(-1,0)$, $(0,1)$, $(0,-1)$. [16]

Q8. (a) State and prove Gauss Divergence Theorem.

(b) Verify Gauss Divergence Theorem for the vector $\vec{A} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ over the region bounded by $x^2 + y^2 + z^2 = 9$. [8 X 2=16]

Exam Code: 112414 **Paper Code: 4206**
(30)

Programme: Bachelor of Science (Honours) Mathematics
Semester-IV

Course Title: Partial Differential Equations

Course Code: BOML-4332

Time Allowed: 3 Hours **Max Marks: 80**

Note: Candidates are required to attempt five question, selecting at least one question from each section. The fifth question may be attempted from any section. Each question carries equal marks.

Section A

1. (a) Find the partial differential equation of all planes at a distance a from the origin. 8

(b) Find the general solution of the linear partial differential equation 8

$y^2p - xyq = x(z - 2y)$.

2 (a) Prove that general solution of lagrange's first order linear partial differential equation $Pp + Qq = R$ is $F(u, v) = 0$. 8

(b) Find the general solution of the linear partial differential equation

$(x + 2z)p + (4zx - y)q = 2x^2 + y.$ 8

Section B

3. (a) Find the complete integral of the equation

$$p^2x + q^2y = z$$ 8

(b) Find the integral surface of the linear partial differential equation

$$x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$$ 8

which contains the straight line $x + y = 0, z = 1$.

4. (a) Find the characteristics of the equation $pq = z$, and determine the integral surface which passes through the parabola $x = 0, y^2 = z$. 8

(b) Solve $(2D^2 - 5DD' + 2D'^2)z = 24(y - x)$ 8

Section C

5. (a) Solve $(D^2 - D'^2 - 3D')z = e^{x+2y}$ 8

(b) Solve $(D - D'^2)z = \cos(x - 3y)$ 8

6. (a) Solve $x^2D^2 - y^2D'^2 + Dx - D'y = \log x$ 8

(b) Solve $p + s - q = xy$ 8

Section D

7 (a) Solve $t + s + q = 0$ 8

(b) Show that a surface of revolution satisfying the differential $r = 12x^2 + 4y^2$ and touching the plane $z = 0$ is $z = (x^2 + y^2)^2$. 8

8.(a) Classify

$$u_{xx} + 2u_{yy} + u_{zz} = 2u_{xy} + 2u_{yz}$$ 8

(b) Solve $rx = (n - 1)p$ 8

Exam Code: 112414 Paper Code: 4207

(30)

Programme: Bachelor of Science (Honours) Mathematics

Semester-IV

Course Title: Group Theory

Course Code: BOML-4333

Time Allowed: 3 Hours **Max Marks: 80**

Note: Attempt five questions in all, selecting at least one question from each section. The fifth question may be attempted from any section. Each question carries equal marks.

Section-A

1. a . Show that $\frac{n(n^2+2)}{3}$ is an integer for all $n \in N$ 5

   b. Use the Euclidean Algorithm to find integers x and y such that g.c.d (1769, 2378)=1769x=2378y 8

   c. Prove that lcm [a,b]=|b| if a|b for non-zero integers a and b 3

2. a. State and prove fundamental theorem of Arithmetic. 8

   b. show that the Relation R in the set A =$\{x \in Z : D \leq x \leq 12\}_9$ given by R = $\{(a,b) : |a-b|$ is multiple of 4 $\}$

is an equivalence relation. Find the set of all elements related to 1 8

Section-B

3. a. Prove that the set Q^+ of positive rationals is a group under ordinary multiplication. 6

b. If G is a group then every a∈G has unique inverse in G 5

c. Let $H = \{ a + bi : a, b \in R, a^2 + b^2 = 1\}$. Prove or Disprove that H is subgroup of group of non- zero complex numbers C* under multiplication. Describe the elements of H geometrically 5

4. a. A non empty subset H of a group G is a subgroup of G iff $ab^{-1} \in H \; \forall a, b \in H$. 10

b. Let x and y belong to group G. if $x\,y \in (G)$, Prove that $xy = yx$. 6

Section-C

5. a. if H and K are two subgroup of a group G, them HK is a subgroup of G iff HK = KH 10

b. Show that the set H= $\left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \in R\ stad - bc = 1\right\}$ is normal subgroup of the group $G = \left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \in R \text{ such that } ad - bc \neq 0\right\}$ 6

6. a. Find all generators of Z_6 w.r.t. addition modula 6. 4

b. Prove that every subgroup of a cyclic group is cyclic. 8

c. Prove that a factor group of an abelian group is abelian. 4

Section-D

7. a. if α= (1 2) (4 5), and $\beta = (1\,6\,5\,3\,2)$ then compute each of the following (a) α^{-1} (b) $\beta\,\alpha$ (c) $\alpha\,\beta$ 8

b. if the pair of cycles α =$(a_1\, a_2 \ldots\ldots\ldots a_m)$ and β =$(b_1\, b_2 \ldots\ldots\ldots b_n)$ have no entries in common, then $\alpha\,\beta = \beta\alpha$. 8

8. a. Prove that every permutation in S_n, n>1, is a product of 2- cycles 8

b. Define alternating group A_n Prove that A_n has order $\frac{n}{2}$ for n>1. 8

Exam Code: 112414 **Paper Code: 4208**

(30)

Programme: Bachelor of Science (Honours) Mathematics

Semester-IV

Course Title: Statistical Methods

Course Code: BOMM-4334

Time Allowed: 3 Hours **Max Marks: 50**

Note: Attempt five questions in all selecting at least one question from each section. Fifth question can be attempted from any section. Each question carries equal marks. Students are allowed to use statistical Tables and a simple non programmable and no-storage calculator

Section -A

Q-1 (a) State and prove all the properties of Karl Pearson Coefficient of Correlation

(b) For two random variables X and Y with common mean , the two regression lines are y=ax+b and X= CY +d. Show that b/d =1-a/1-c (5,5)

Q-2 Obtain the equation of two lines of regression for the following data

X: 65 66 67 67 68 69 70 72

Y: 67 68 65 68 72 72 69 71 10

Section-B

Q-3 (a) If X_1 *and* X_2 are two independent χ^2 variates with n_1 *and* n_2 degrees of freedom respectively

Then Prove that $\frac{X_1}{X_2}$ is a $\beta_2\left(\frac{n_1}{2},\frac{n_2}{2}\right)$ variate

(b) Derive limiting form of t- Distribution for large degrees of freedom (5,5)

Q-4 (a) if F~F (n_1,n_2) distribution .If $n_2 \to \infty$, then show that $\chi^2 = n_2 F$ follows chi- square distribution *with* n_1 degrees of freedom

(b) State all the properties of t-distribution and prove that m.g.f of t-distribution does not exist (5,5)

Section-C

Q-5 (a) Explain in detail one tailed and two tailed test for Z- test with the help of example and diagram.

(b) A machine put out 20 imperfect articles in a sampir of 600. After the machine is overhauled, it puts out 6 imperfect articles in a batch of 200. Has the machine been improved? (5,5)

Q-6 a) Show how you would use student's t-test to decide whether the two sets of observations:

17 27 18 25 27 29 27 23 17

And 16 16 20 16 20 17 15 21

Indicate that samples drawn from the same universe.

b) A certain drug was given to each of the 10 patients which resulted in a following decrease in blood pressure:

8, 5, 7, 10 ,-1, 0 ,4, 6 ,12, 11

Can we conclude that the drug be used for controlling blood pressures (5,5)

Section-D

Q- 7 A survey of 800 families with four children each revealed the following distribution:

No. of Boys	0	1	2	3	4
No. of Girls	4	3	2	1	0
No. of families	32	178	290	236	64

Test the fitness of Binomial distribution given that male and female births are equally probable. 10

Q-8 a) Show that for a 2×2 contingency table:

a	b
c	d

The value of χ^2 for testing independence of attribute is $\frac{(a+b+c+d)(ad-bc)^2}{(a+b)(c+d)(a+c)(b+d)}$

Q -8 (b)Define F-Statistics. Write down its sampling distribution:Two independent samples of 8 and 7 items are given below:

Sample I	39	41	43	41	45	39	42	44
Sample II	40	42	40	44	39	38	40	

Do the population variance differs significantly. Test at 5% level of significance (5,5)

Exam Code: 112414 **Paper Code: 4209**
(30)

Programme: Bachelor of Science (Honours) Mathematics
Semester-IV

Course Title: Foundation of Statistical Computing

Course Code: BOMM-4135

Time Allowed: 3 Hours **Max Marks: 50**

NOTE: Candidates are required to attempt five questions. selecting at least one question from in each section. The fifth question may be attempted from any section. Each question carries equal marks.

Section-A CO1

1. What are vectors in R? Explain in detail various vector operations available in R. 10
2. Discuss the usage of lists in R. Write R script to create a list with three different objects; a vector, a matrix and a data frame. Print all the elements of the created list. Also, merge the list elements and print them. 10

Section —B CO2

3. Which matrix operations are applicable to data frames? Explain with examples. 10

4. How strings are manipulated in R? Explain any five string functions for creation and modifications of strings in R. 10

Section- C CO3

5. a) Differentiate between read.csv() and Write.csv() functions in R. 5

   b) Create a user defined function fact(i) that returns the factorial of i using functions. 5

6. Discuss the control structures with looping statements in R giving suitable examples. 10

Section-D CO4

7. Describe scatter plot and Box plot available in R to visualize the data. Also, explain the purpose of each plot. 10

8. What are the different ways to customize a plot in R? Write R script to create a legend that explains different box types and customizes it using graphical parameters. 10