11/12/18 EVE

Exam. Code : 211001
Subject Code : 3836

M.Sc. Mathematics Ist Semester

MATH-553 ALGEBRA–I

Time Allowed—3 Hours] [Maximum Marks—100

Note :— Candidates are required to attempt **five** questions, selecting at least **one** question from each Section. The **fifth** question may be attempted from any Section.

SECTION—A

1. (a) Let H and K be two subgroups of a group G. Then HK is subgroup of G if and only if HK=KH. 6

   (b) The intersection of two subgroups of finite index is of finite index. 4

   (c) State and prove Lagrange's theorem and prove that for every $a \in G$, $o(a) \mid n$, where n is order of G. 10

2. (a) Every cyclic group is isomorphic to $\mathbb{Z}$ or to $\frac{\mathbb{Z}}{\langle n \rangle}$ for some $n \in \mathbb{N}$. 7

   (b) Give an example of a group G having subgroups K and T such that K is normal in T and T is normal in G but K is not a normal subgroup of G. 3

(c) Prove that a non-abelian group of order 6 is isomorphic to S_3. 10

SECTION—B

3. (a) Show that each dihedral group is homomorphic to the group of order 2. 5

(b) Find Aut(K) where K is the Klein four-group. 5

(c) If a permutation $\sigma \in S_n$ is a product of r transpositions and also a product of s transpositions, then r and s are either both even or both odd. 10

4. (a) Show that A_n is simple for all $n \geq 5$. 10

(b) Show that the group $\mathbb{Z}_8$ cannot be written as the direct sum of two nontrivial subgroups. 5

(c) Prove that there is a 1-1 correspondence between the family F of nonisomorphic abelian groups of order p^e, p prime and the set P(e) of partitions of e. 5

SECTION—C

5. (a) Let G be a group containing an element of finite order n > 1 and exactly two conjugacy classes. Prove that | G | = 2. 7

(b) State and prove Jordan-Holder theorem. 7

(c) Let G be a group of order 108. Show that there exist a normal subgroup of order 27 or 9. 6

6. (a) State and prove Sylow's second theorem. 7

(b) Let G be a finite group of order p^n, where p is prime and n > 0. Then prove that $Z \cap N$ is nontrivial for any nontrivial normal subgroup N of G. 7

(c) Show that a simple group is solvable if and only if it is cyclic. 6

SECTION—D

7. (a) Find all ideals in $\mathbb{Z}$ and also in $\mathbb{Z}_{10}$. 5

(b) If R is a ring with unity, then each maximal ideal is prime. Is converse true ? Justify. 6

(c) Let F be a field. Then characteristic of F is either 0 or a prime number p. 5

(d) Define idempotent and find the idempotents of ring $\mathbb{Z}_{12}$. 4

8. (a) Show that there exist a ring homomorphism $f : \mathbb{Z}_m \to \mathbb{Z}_n$ if and only if n | m. 6

(b) Prove that the ideal $<x^3 + x + 1>$ in the polynomial ring $\mathbb{Z}_2[x]$ over $\mathbb{Z}_2$ is a prime ideal. 6

(c) Define integral domain and show that a finite integral domain is a division ring. 8

Exam. Code : 211001
Subject Code : 3837

M.Sc. Mathematics 1st Semester

MECHANICS—I

Paper—MATH-554

Time Allowed—Three Hours] [Maximum Marks—100

Note :— Attempt **FIVE** questions in all, selecting at least **ONE** question from each Section. All questions carry equal marks.

SECTION—A

1. Obtain the radial and transverse components of acceleration of a particle which describe the plane curve r = f(θ) in the form :

$$\ddot{r} - r\dot{\theta}^2, \frac{1}{r}\frac{d}{d\theta}(r^2\dot{\theta}).$$

If the curve is the equiangular spiral $r = a \exp(\theta \cot \alpha)$ and if the radius vector to the particle has a constant angular velocity, show that the resultant acceleration of the particle makes an angle 2α with the radius vector and its magnitude $\frac{v^2}{r}$, where v is the speed of the particle.

2. (a) Show that two equal and opposite rotations of a rigid body about distinct parallel axes are equivalent to a translation of the body.

(b) A rigid lamina is moving in its own plane and one point A in it has velocity $\vec{u}$ relative to a fixed origin 0. If $\vec{w}$ is the angular velocity of the lamina, show that the point P in it has velocity $\vec{u}+\vec{w}\times\vec{r}$ relative to 0, where $\vec{r}=\overrightarrow{AP}$. Hence or otherwise prove that the position vector $\vec{r}\,'$ of the instantaneous centre I of the lamina relative to A is given by $\vec{r}\,' = (\vec{w}\times\vec{u})/w^2$.

SECTION—B

3. (a) Explain the Principle of conservation of energy for a single particle.

(b) Define the impulse of a force over a finite time interval and derive the equation of impulsive motion of a particle. Also show that the K.E. gained is $\frac{\vec{I}}{2}(\vec{v}_1+\vec{v}_2)$ where an impulse $\vec{I}$ changes the velocity of a particle of mass m from $\vec{v}_1$ to $\vec{v}_2$.

4. (a) Show that the acceleration of a particle P moving along a plane curve c is $\ddot{s}\hat{t}+(\dot{s}^2/\rho)\hat{n}$, where s denotes the arc length along c, $\hat{t}, \hat{n}$ are unit vectors along the tangent and normal at P respectively and ρ is the radius of curvature at P.

(b) Show that the rate of increase of angular momentum about the axis is equal to the moment of the resultant force which acts on the particle.

SECTION—C

5. (a) Derive an expression for the differential equation of a particle moving in a central orbit in pedal co-ordinates.

(b) If $P = \mu(u^2 - au^3)$, where $a > 0$ and a particle is projected from an apse at a distance a from the centre of force with a velocity $(\mu c/a^2)^{1/2}$, where $a > c$, prove that the other apsidal distance of the orbit is $a(a+c)/(a-c)$ and find the apsidal angle.

6. (a) A fixed nucleus S, having positive charge, Ze repels a particle P having mass m and positive charge e'. P is projected from a great distance at infinity with initial speed v_0 in a direction whose perpendicular distance from S is d, the medium being a vacuum. Show that its ultimate direction of motion makes an angle ϕ with an initial direction where $\cot\frac{\phi}{2}$ $(dm\ v_0^2/Zee')$.

(b) A comet travelling in an elliptic orbit round the sun under an attraction μ/r^2 per unit mass has its tangential velocity increased a small amount δv. Taking 2a to be the major axis and e the eccentricity of the former orbit, show that the comet's least distance from the sun is increased by

$$4\,4\delta v[a^3(1-e)/\mu(1+e)]^{1/2}.$$

SECTION—D

7. (a) State and prove the parallel axes theorem for moments of inertia and for products of inertia for a system of particles.

(b) A square of side 2a has particles of masses m, 2 m, 3 m, 4 m at its vertices. Find the principal moments of inertia at the centre of the square and also the directions of the principal axes.

8. (a) Define equimomental systems. Show that a solid cuboid of mass M is equimomental with masses $\frac{M}{24}$ at the mid points of its edges and $\frac{M}{2}$ at its centre.

(b) In coplanar distribution, show that the M.I. attains extreme values along the principal axes through O in the plane of distribution.