27-11-18

Exam Code: 211001 **Paper Code: 8352 (30)**

Programme : M.Sc. (Maths) Semester-I

Course Title: Real Analysis-I

Course Code: MMSL-1331

Time Allowed: 3 Hours **Max Marks: 80**

Note : There are four sections in paper and candidates are required to attempt five questions, selecting at least one question from each section. The fifth question may be attempted from any section. Each question carries equal marks.

Section A

1. Prove that every K-cell is compact.

2. (a) Prove that compact sets are closed.

(b) Prove that in a metric space every open ball is an open set.

Section B

3. State and prove Cantor's Intersection Theorem.

4. (a) Prove that closure of connected set is connected.

(b) Compact metric space is always complete.

Section C

5. State and prove Baire's category theorem.

6. (a) Prove that continuous image of connected set is connected.

(b) If f : X →Y is a Mapping, Then f is continuous iff

$$f(\bar{A}) \subseteq \overline{f(A)} \quad \forall A \subseteq X$$

Section D

7. State and prove 2nd mean value Theorem of Riemann Stieltje's integral.

8. (a) If F is monotonic on [a,b] and if α is continuous on [a, b] then

F ∈ R (α) on [a,b]

(b) State and prove fundamental theorem of calculus.

Exam Code: 211001 **Paper Code: 8353** **(30)**

Programme: M.Sc. (Maths) Sem: I

Course Title: Complex Analysis

Course Code: MMSL-1332

Time Allowed: 3 Hours **Max Marks: 80**

Note:

There are eight questions of equal marks. Attempt any five selecting atleast one from each section.

The fifth question may be selected from any section. Each question carries 16 marks

UNIT I

1(a) Define analytic function along with examples . State and prove necessary condition for a function to be analytic. **(10)**

(b) If v=sinhxcosy, determine the conjugate function u. **(6)**

2(a) Show that for the function **(10)**

$$f(z)=\begin{cases} \frac{x^3(1+i)-y^3(i-i)}{x^2+y^2}, & z \neq 0 \\ 0, & z=0 \end{cases}$$

Cauchy Reimann equations are satisfied at origin yet f(z) is not differentiable .

(b)If f(z)=u+iv , z=x+iy , prove that **(6)**

$$\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}=4\frac{\partial^2 f}{\partial z\partial \bar{z}}$$

UNIT-II

1(a)State and prove Liouville's Theorem. **(10)**

(b)Find the bilinear transformation which maps the points ∞,i,0 in the z plane to the points 0,i,∞ in the w plane. **(6)**

2(a)Find the bilinear transformation which maps the unit circular disc |z|≤1 in the z plane onto the unit circular disc | w|≤1 in the w plane. **(10)**

(b)Find the fixed points of the transformation w=$\frac{z}{z-2}$ and find the normal form . **(6)**

UNIT –III

1(a)State and prove taylors theorem. **(10)**

(b)Find the radius of convergence of the series $\sum \frac{n!^2 z^n}{(2n)!}$ **(6)**

2(a)State and prove Rouches Theorem. **(10)**

(b)Prove that all the roots of $z^7 - 5z^3 + 12 = 0$ **lie between the circles** $|z| = 1 \, and \, |z| = 2$ **(6)**

UNIT IV

1(a)State the prove Cauchy Residue Theorem. **(10)**

(b)Classify the singularity in each case (i) f(z)= $\frac{z - \sin z}{z^3}$ **, z=0** **(6)**

(ii) f(z)= $z^2 e^{\frac{1}{z}}$ **,z=0**

2(a)Evaluate $\int_0^\infty \frac{\cos mx}{x^2 + a^2} dx$ **(10)**

(b)Evaluate $\int_0^{2\Pi} \frac{d\theta}{2 + \cos\theta}$ **(6)**

M.Sc. Mathematics Semester-I

Course Title **Algebra-I**

Course code: MMSL-1333

Time: 3Hrs **Max. Marks: 80**

Instructions: Attempt five questions in all, selecting at least one question from each section. The fifth question may be attempted from any section.

Section-I

1 (i) If H is a subgroup of G, then centralizer of H i.e. $C(H) = \{x \in G: xh = hx\ , for\ all\ h \in H\}$. Prove that $C(H)$ is a subgroup of G. (4)

(ii) State and prove Lagrange's theorem. (6)

(iii) Let $G =< a >$ be a cyclic group of order *n*, show that $G =< a^k >$ if and only if gcd $(k, n) = 1$. (6)

2. (i) Show that U_9 is a cyclic group. What are all its generators? (4)

(ii) Let G be a group and let Z(G) be the centre of G. If G/Z(G) is cylic, then prove that G is abelian group. (5)

(iii) If H and K are two subgroups of a group G. Prove that H ∪K is a subgroup of G if and only if either H ⊆K or K ⊆H. (7)

Section-II

3. (i) Let ϕ be a homomorphism of G onto $\bar{G}$ with kernel K and let $\bar{N}$ be normal subgroup of $\bar{G}$. If $N = \{x \in G: \phi(x) \in \bar{N}\}$, then prove that $G/N \approx \bar{G}/\bar{N}$. (6)

(ii) State and prove Cayley's theorem. (6)

(iii)Show that there are two abelian groups of order 108 that have exactly four subgroups of order 3. (4)

4. (i) Prove that A_4 has no subgroup of order six. (4)

(ii) Prove that every permutation of a finite set can be written as a cycle or as a product of disjoint cycles. (5)

(iii) Let G and H be finite cyclic groups. Prove that G ⊕ H is cyclic if and only if |G| and |H| are relatively prime. (7)

Section-III

5. (i) Let G be a finite group. Prove that the number of Sylow p- subgroups in G is equal to 1 modulo p and divides |G|. (7)

(ii) Let $|G| = 2p$, where p is an odd prime, prove that G is isomorphic to Z_{2p}. (4)

(iii) State and prove Cauchy's theorem. (5)

6. (i) Let a group G is solvable and N is a normal subgroup of G, prove that G/N is also solvable group. (5)

(ii) State and prove Jordan Holder theorem. (7)

(iii) Find all composition series of $S_3 \times Z_2$. (4)

Section-IV

7. (i) Suppose that R is a ring and that $a^2 = a$ for every $a \in R$, show that R is commutative. (5)

(ii) Let $\phi: R \to R'$ is a ring homomorphism, if R is commutative, then prove that $\phi(R)$ is also commutative. (5)

(iii) Let $\phi: R \to S$ is a ring homomorphism, prove that the mapping $\psi: R/Ker\,\phi \to \phi(R)$, defined by $\psi(r + Ker\,\phi) = \phi(r)$ is an isomorphism. (6)

8. (i) Prove that the set S of all matrices of the form $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ with $a, b \in Z$, forms a subring of $M_2(Z)$. Prove that further that S is neither a left nor right ideals of $M_2(Z)$. (4)

(ii) Let R be a commutative ring with unity and let A be an ideal or R , prove that R/A is an integral domain if and only if A is prime. (7)

(iii) Give an example of a ring in which some prime ideal is not maximal. (5)

Exam Code: 108201 **Paper Code: 8355 (20)**

Programme: M.Sc. (Mathematics) Sem-I

Course Title: Mechanics-I

Course Code: MMSL-1334

Time Allowed: 3 Hours **Max Marks: 80**

Instructions:

Candidates are required to attempt five questions in all selecting at least one question from each section.

The Fifth question may be attempted from any section.

Section-A

1. Obtain the expression for the velocity and acceleration of a particle in case of moving axes and hence deduce the result for cylindrical and spherical coordinates. 16

2. P is the point specified by polar coordinates (r,θ) in a plane and its position vector $\vec{OP}$ at any time t with respect to fixed origin O is represented by the complex number $z = re^{i\theta}$. Show that $\dot{z} = \dot{r}e^{i\theta} + r\dot{\theta}(ie^{i\theta})$, $\ddot{z} = (\ddot{r} - r\dot{\theta}^2)e^{i\theta} + r\ddot{\theta} + 2\dot{r}\,\dot{\theta})(ie^{i\theta})$ and hence deduce the radial and transverse velocity and acceleration components of P. 16

Section-B

3. a) State and prove principle of conservation of energy. 8

b) A smooth wire bent in the form of the parabola is fixed with its axis vertical and vertex downwards. A particle of mass m oscillates on the wire coming to rest at the extremities of the latus rectum. Show that the reaction of the wire on the particle when passing through the vertex is 2mg. 8

4. Discuss shot putt analysis in particle dynamics. 16

Section-C

5. Establish the formulas $\frac{d^2U}{d\theta^2} + U = \frac{P}{h^2\,\theta^2}$, $\dot{\theta} = hU^2 (U = \frac{1}{r})$ for the motion of a particle describing a central orbit under an attraction P per unit mass. If $P = \mu u^5$, find the speed v with which the particle can describe the circle r=a. If the particle moves under this attraction with the same areal constant as in the circular path, and $\dot{r} = -\frac{3v}{4\sqrt{2}}$ when r=2a, $\theta = 0$, find the equation of the spiral path of the particle and show that as $\theta \to \infty$ the path is asymptotic to the circle r=a. 16

6. a) Discuss Kepler's Laws of planetary motion. 8

b) Derive pedal equation of ellipse in the form

$$\frac{b^2}{p^2} = \frac{2a}{r} - 1$$ 8

Section-D

7. Find the greatest and least Moment of Inertia for a coplanar distribution in which the lines through origin making an angle θ and $\frac{\pi}{2} + \theta$ are taken as principle axes. 16

8. a) A uniform rigid rod AB moves so that A and B have velocities V_A, V_B at any instant. Show that the Kinetic energy is $T = \frac{1}{6} M(V_A^2 + V_A.V_B + V_B^2)$, M being the mass. 8

b) Find an equimomental system of particles for a uniform rod AB of mass M. 8

Exam Code: 211001 **Paper Code: 8356 (30)**

Programme: M.Sc. (Maths) Sem: I

Course Title: Differential Equations

Course Code: MMSL- **1335**

Time Allowed: 3 Hours **Max Marks: 80**

Note

There are eight questions of equal marks. Attempt any five selecting atleast one from each section.

The fifth question may be selected from any section. Each question carries 16 marks.

UNIT I

1(a)Find third order approximation of the solution of equation $\frac{dy}{dx}=x+y^2$ **; y(0)=0** **(10)**

(b)Solve the equation $\frac{dx}{z(x+y)}=\frac{dy}{z(x-y)}=\frac{dz}{x^2+y^2}$ **(6)**

2(a)Find the eigen values and eigen functions of the Strum Liouville problem $y''+\lambda y=0$ **(10)**

$y'(0)=y'(\pi)=0$

(b)Find the adjoint of $x^2\frac{d^2y}{dx^2}+(2x^3+1)\frac{dy}{dx}+y=0$ **(6)**

UNIT II

1(a)Using the Laplace Transform solve the differential equation $\frac{d^2x}{dt^2}+2\frac{dx}{dt}+5x=e^{-t}\sin t$ **(10)**

(b)Evaluate $L^{-1}\left[\frac{p^2}{(p^2+a^2)(p^2+b^2)}\right]$ **(6)**

2(a)State the Heaviside expansion formulae and hence evaluate $L^{-1}\left[\frac{1+2p}{(p+2)^2(p-1)^2}\right]$ **(10)**

(b) Evaluate $L\left(e^{-t}t\cos 2t\right)$ **(6)**

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UNIT III

1(a)Define fourier transform. Find the fourier transform of **F(x)=** $\begin{cases} 1 & |x| < a \\ 0 & |x| > a \end{cases}$ **(10)**

and hence prove $\int_0^\infty \frac{\sin sds}{s} = \frac{\pi}{2}$

(b)If f(s) is the fourier transform of function f(x) then F(f(x)cosax)= $\frac{f(s+a)+f(s-a)}{2}$ **(6)**

2(a)Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ $\textit{given that } u(0,t) = u(\pi,t) = 0$ **and u(x,0)=2x where** $0<x<\pi, t>0$ **(10)**

(b) Find the finite fourier sine transform of f(x)= x^2 **,** $0<x<4$ **(6)**

UNIT IV

1)aProve that $\int_{-1}^{1} P_m(x)P_n(x)dx = \begin{cases} \frac{2}{2n+1} & if\ m=n \\ 0 & if\ m \neq n \end{cases}$ **(10)**

b) Prove that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$ **(6)**

2(a) $H_n(x) = 2^n \exp\left(\frac{-1}{4}\frac{d^2}{dx^2}\right)x^n$ **(10)**

(b) $(n+1)L_{n+1}(x) = (2n+1-x)L_n(x) - nL_{n-1}(x)$ **(6)**