Exam. Code : 209001
Subject Code : 3751

M.Sc. Physics 1st Semester

PHY-402 : MATHEMATICAL PHYSICS

Time Allowed—3 Hours] [Maximum Marks—100

Note :— The question paper has ***four*** Sections (A-D). Attempt ***five*** questions selecting at least ***one*** question from each section. The fifth question may be attempted from any section.

SECTION—A

1. (a) Plot graph of the following function :

$f(x) = x \quad 0<x<4$

$= -x, \quad -4<x<0$, period of function = 8 5

(b) Develop the Fourier cosine expansion of $f(x) = -x$ in the half interval (0, L). 10

(c) Find the Fourier transform of

$f(x) = 1/\varepsilon \quad |x| < 1$

$= 0 \quad |x| > 1$ 5

2. (a) Define symmetric, anti-symmetric and mixed tensors. 5

(b) Find the expression of a curl of a vector function in cylindrical co-ordinates. 10

(c) By use of summation convention rewrite :

$$d\phi = \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy + \frac{\partial \phi}{\partial z} dz$$ 5

SECTION—B

3. (a) Using Frobenius method solve the differential equation :

$$x^2\frac{d^2y}{dx^2}+6x\frac{dy}{dx}+(6-x^2)y=0.$$ 10

(b) Write down the expression for generating function of Bessel function, $J_n(x)$. Use it to prove that :

$$\frac{d}{dx}(J_n(x)) = \frac{1}{2}(J_{n-1}(x) + J_{n+1}(x))$$ for the case of integer, n. 10

4. (a) Consider the Laplace equation $\Delta^2 u = 0$ in spherical coordinates, assume $u = R\Theta$, where R depends only on r and Θ only on θ, use the method of separation of variables to obtain two equations only in r and θ variables. What are the solutions of equation of R known as ? 12

(b) Define Gamma function, $\Gamma(n)$, show that :

$$1.3.5.7.....(2n-1) = \frac{2^{1-n}\Gamma(2n)}{\Gamma(n)}$$ 8

SECTION—C

5. (a) State the prove Cauchy residue theorem. 10

(b) Classify the singularities and calculate the residue for $F(z) = z.\exp(1/z)$. 5

(c) By use of the residue theorem, evaluate :

$$\int_0^{2\pi}\frac{d\vartheta}{3-2\cos\vartheta+\sin\vartheta}.$$ 5

6. (a) Show that $\int_{z_o}^{z} Z^n dz = \frac{Z^{n+1}-Z_o^{n+1}}{n+1}$

for all n, except n = –1. Discuss the case for n = –1. 10

(b) By use of residue theorem, evaluate :

$$I = \int_{-\infty}^{\infty}\frac{\sin x}{x}dx$$ 5

(c) Find the Laurent expansion for $F(Z) = \frac{1}{Z(Z-2)}$ in the region $0<|z|<2$. 5

SECTION—D

7. (a) Show that the following sets are groups under the given laws of composition :

(i) The set of all m×n matrices under matrix addition.

(ii) The set of all non-zero rational numbers under scalar multiplication. 10

(b) Define permutation group and give one example. Discuss their importance in quantum mechanics of identical particles. 10

8. (a) Explain what is Isomorphism and Homomorphism. 10

(b) Consider the following four operations in the XY plane :

(i) No change $\{x \rightarrow x, y \rightarrow y\}$

(ii) Inversion $\{x \rightarrow -x, y \rightarrow -y\}$

(iii) Reflection $\{x \rightarrow -x, y \rightarrow y\}$

(iv) Reflection $\{x \rightarrow x, y \rightarrow -y\}$

Prove that these four operations form a mathematical group. 10