Exam. Code : 206702
Subject Code : 4619

M.Sc. (Computer Science) 2nd Semester

THEORY OF COMPUTATION

Paper : MCS-201

Time Allowed—Three Hours] [Maximum Marks—100

Note :— Attempt any **FIVE** questions. All questions carry equal marks.

1. Derive a grammar for odd length palindromes generated over $\Sigma = \{0, 1\}$. Hence convert it to Chomsky Normal form.
2. Give a regular expression for representing strings generated over $\Sigma = \{0, 1\}$ ending with 01. Give also corresponding regular grammar.
3. What is Kuroda Normal Form ? Give a grammar in that form ? What type of grammar it will be ?
4. Design an automata having one self loop and at least two final states. Write the grammar corresponding to the automata.
5. Design a PDA for accepting even length palindrome generated over $\Sigma = \{2, 3\}$.

6. Design a Turing machine to add two numbers.
7. Describe the formal properties of LL(k) grammars.
8. Write short notes on any **TWO** of the following :—
   (a) Closure properties of a grammar
   (b) Derivation Graph
   (c) Rewriting system.

Exam. Code : 206702
Subject Code : 4620

M.Sc. Computer Science Semester—II

MCS–202 : IMAGE PROCESSING

Time Allowed—3 Hours] [Maximum Marks—100

Note :– Attempt any **five** questions. Each question carries equal marks.

1. Define the term Image Processing. Explain different steps used in Image Processing. Also discuss different available image data formats. 20

2. What do you understand by Visual Phenomena ? Discuss. 20

3. What do you understand by Image Data Compression ? Explain different techniques used for Image Data Compression. Discuss Pixel Coding technique in detail. 20

4. What do you mean by Image Enhancement ? Discuss different techniques used for Image Enhancement. Explain at least one in detail. 20

5. (a) What are various components of General Purpose Image Processing System ? Explain the role of each component. 10

   (b) Discuss the process of Image Digitization. 10

6. Discuss Digital Image Restoration System. Enlist Digital Image Restoration models. Explain the concept of Linear Filtering model. 20

7. Write short notes on the following :

   (a) Color Models 10

   (b) Color System Transformation. 10

8. Discuss the applications of Image Processing in the field of Medical Image Processing. 20

Exam. Code : 209002
Subject Code : 4770

M.Sc. (Physics) Semester—II

Phy–451 : QUANTUM MECHANICS—I

Time Allowed—3 Hours] [Maximum Marks—100

Note :—Section A is compulsory. Attempt at least **ONE** question each from Sections B, C, D and E.

SECTION—A

1. (a) Find the eigen values and eigen vectors of a projection operator.

(b) Prove that $-i\hbar\frac{d}{dt}$ is the operator that represents momentum.

(c) Prove that the product of two Hermitian operators A and B is Hermitian only if [A, B] = 0.

(d) Find eigen values for $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$.

(e) Show that $|\phi> = \sum_i c_i u_i$ is equivalent to $\sum_i |u_i><u_i| = 1$.

(f) Prove $\hat{U}^{-1}(t, t_0) = \hat{U}(t_0, t)$.

(g) Find $[P_x, X]$, where P_x is the momentum operator in X direction.

(h) Consider a 3d state. Construct the state of the total angular momentum J = 5/2 and M_J = 5/2. Calculate Clebsh-Gordon coefficient <22j 1/2 1/2 | 5/2 5/2>.

(i) Find $[a, a^+]$ where a and a^+ are the annihilation and creation operators for a Harmonic oscillator.

(j) Consider the wave function $\psi(x, t) = A\, e^{-\lambda|x|}\, e^{-i\omega t}$, where A, λ and ω are positive real constants. Normalize ψ. $2\times10=20$

SECTION—B

2. (a) To every ket corresponds a bra, is it possible to find bras which have no corresponding ket ? Support your answer in detail with at least two examples. 10

(b) Consider a physical system with a three-dimensional state space. An orthonormal basis of the state space is chosen in this basis the Hamiltonian is represented by the matrix $H = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. What are the possible results when the energy of the system is measured ? A particle is in the state $|\psi>$, represented in this basis as $\frac{1}{\sqrt{3}}\begin{bmatrix} i \\ -i \\ i \end{bmatrix}$. Find $\langle H \rangle$, $\langle H^2 \rangle$ and ΔH. 10

3. (a) How is transformation matrices used for changing basis ? Discuss in detail. 10

(b) An operator $\hat{A}$, representing an observable A, has two normalized eigenstates ψ_1 and ψ_2 with eigen-values a_1 and a_2 respectively. Another operator B, representing an observable B, has two normalized eigenstates ϕ_1 and ϕ_2, with eigen values b_1 and b_2, respectively. The eigenstates of these two operators are related by

$$\psi_1 = (3\phi_1 + 4\phi_2)/5$$

$$\psi_2 = (4\phi_1 - 3\phi_2)/5$$

Now, observable A is measured and the value a_1 is obtained. What is the state of the system immediately after the measurement ? After the measurement, suppose now B is measured, what are the possible results and what are their probabilities ? 10

SECTION—C

4. (a) Differentiate between the time development of quantum mechanical system in the Schrödinger and Heisenberg representation. 10

(b) Assume that in the Schrödinger picture all operators are time-independent. Work in the Heisenberg picture and derive an equation expressing the time evolution of operator $A_H(t)$. 10

5. (a) Find the differential and integral equation of motion for $\hat{U}$. Generalise the equation when $\hat{H}$ does not depend explicity on time. 10

(b) State and prove the Ehrenfest's theorem. 10

SECTION—D

6. Consider a one-dimensional harmonic oscillator with Hamiltonian

$$H = \frac{\hbar\omega}{2}(Q^2 + P^2)$$

where $\hat{P} = \frac{P}{\sqrt{m\omega\hbar}}$ and $\hat{Q} = x\sqrt{\frac{m\omega}{\hbar}}$

(a) Compute the commutation relation $[\hat{P}, \hat{Q}]$.

(b) For raising and lowering operators a^+ and a defined as

$$a^+ = \frac{1}{\sqrt{2}}[\hat{Q} - i\hat{P}] \text{ and } a = \frac{1}{\sqrt{2}}[\hat{Q} + i\hat{P}]$$

compute $a|n>$ and $a^+|n>$, where $|n>$ is the eigen function of the oscillator for the n^{th} energy state.

(c) Compute the matrix elements of operator :

$x\ (x_{n_k} = <n|\,x\,|k>)$. 5+10+5

7. (a) Consider a particle of mass m confined in an infinite one-dimensional potential well of width a

$$V(x) = \begin{cases} 0 & -a/2 \le x \le a/2 \\ \infty & \text{otherwise} \end{cases}$$

Find the eigenstates of the Hamiltonian (i.e. the stationary states) and the corresponding eigen energies. 13

(b) Find first excited state of a Harmonic oscillator. 7

SECTION—E

8. (a) Two angular momenta of respective magnitudes j_1 and j_2 and total angular momentum $J = j_1 + j_2$ are described by the basis $|m_1\ m_2> = |j_1\ m_1> \otimes |j_2\ m_2>$. By construction, the states $|m_1\ m_2>$ are eigen states of $\{j_1^2, j_2^2, J_{1z}, J_{2z}\}$ and $J_z = J_{1z} + J_{2z}$.

Find the eigen values of the operator J_z and their degree of degeneracy. Consider the states :

$$|\psi_+> = |m_1 = j_1,\ m_2 = j_2>$$

$$|\psi_-> = |m_2 = -j_1,\ m_2 = -j_2>$$

for which m_1 and m_2 both assume either maximal or minimal values. Show that the state $|\psi_+>$ and $|\psi_->$ are eigenstates of J^2 and find the corresponding eigen values. 15

(b) Find the eigen function and eigen values of a two-dimensional isotropic harmonic oscillator. Find the degeneracy of the energy levels. 5

9. (a) Consider a particle with L = 1 and S = 1/2, construct all possible eigenstates of total J^2 and J_z from eigenstates $|LM_L>$ and $|SM_S>$ of L^2, L_z and S^2, S_z. Find all non-zero Clebsch-Gordan coefficients $<LM_{Lj}\ SM_S\ |\ J\ M_J>$. 16

(b) Find $[L_x, L_y]$ and $[L^2, L_z]$. The angular momentum $\vec{L}$ is defined by $\vec{L} = \vec{r} \times \vec{p}$. 4

Exam. Code : 206702
Subject Code : 4622

M.Sc. (Computer Science) Semester—II

MCS–204 : FORMAL SPECIFICATION AND VERIFICATION

Time Allowed—3 Hours] [Maximum Marks—100

Note :— (1) There are total **EIGHT** questions. Candidates are required to attempt any **FIVE** questions. All questions carry equal marks.

(2) The students can use only non-programmable and non-storage type calculator.

1. What is the role of formal specification languages ? Discuss the common features of formal specification languages. 20

2. Compare First Order Logic (FOL) with the propositional logic by discussing the pros and cons of FOL and propositional logic. Discuss the terms and predicates of FOL. What is universal and existential quantification ? 20

3. Discuss how Hoare logic can be extended to deal with the languages involving advanced constructs such as procedures with parameters, non-determinism, concurrency, communication and fairness. 20

4. What type of logical errors can occur in formal specifications ? Discuss any two techniques for detecting errors in formal specifications. What is the relationship (if any) between FOL and formal specifications ? 20

5. (a) What is Dijkstra's weakest pre-condition semantics ? What is strongest post-condition ?

   (b) What is the need of data refinements ? Discuss data refinement with the help of an example. 10+10

6. What are the safety and liveness properties ? How specification and verification of reactive programs is done ? 20

7. What is the use of deductive and model-theoretic approaches ? Explain these approaches. 20

8. Write short notes on the following :

   (a) Hoare logic to prove correctness of factorial of number program

   (b) Stack and Queue as abstract data types. 10+10