16. Let g is integrable over E and $< f_n >$ is a sequence of measurable functions such that $| f_n | \leq g$ on E and $f_n \to f$ a.e. on E, then prove that

$$\int_E f = \lim_{n\to\infty} \int_E f_n .$$ 10

UNIT—V

17. If f is a function of bounded variation on [a, b] and a < c < b, then prove that f is of bounded variation in [a, c] and [c, b]. Also $\overset{b}{\underset{a}{T}}(f) = \overset{c}{\underset{a}{T}}(f) + \overset{b}{\underset{c}{T}}(f)$. 10

18. Let $f : [0, 1] \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} 0, & \text{if } x \text{ is rational point of } [0,1] \\ 1, & \text{if } x \text{ is irrational point of } [0,1] \end{cases}$$

find four Dinni's derivatives. 10

19. What do you mean by saying f is absolutely continuous function on [a, b] ? Prove that if f is integrable on [a, b] and $F(x) = \int_a^x f(t)dt$, then F is absolutely continuous on [a, b]. 10

20. If f is bounded and measurable function on [a, b] and

$$F(x) = \int_a^x f(t)dt + F(a)$$

prove that F is differentiable a.e. and F' = f a.e. in [a, b]. 10

Exam. Code : 211002
Subject Code : 4928

M.Sc. Mathematics 2nd Semester

REAL ANALYSIS—II

Paper—MATH-561

Time Allowed—Three Hours] [Maximum Marks—100

Note :— Attempt *two* questions from each unit. Each question carries equal marks.

UNIT—I

1. What do you mean by sequence of functions ? For the sequence $< f_n >$ defined by

$$f_n(x) = \frac{x^n}{1+x^n}, x \in [0, \infty), n \in \mathbb{N}$$

find $f(x) = \lim_{n\to\infty} f_n(x)$. Is f continuous ? 8+2

2. Show that the sequence $< f_n >$ where $f_n(x) = nxe^{-nx^2}$ is not uniformly convergent on [0, k] (k > 0). 10

3. State and prove Dinni's theorem. 10

4. Show that the series for which n^{th} partial sum is

$$s_n(x) = \frac{1}{1+nx}$$

can be integrated term-by-term in [0, 1], however, it is not uniformly convergent on [0, 1]. 7+3

UNIT—II

5. If $< E_n >$ is an infinite increasing sequence of measurable sets, prove that $m(\lim_{n\to\infty} E_n) = \lim_{n\to\infty} m E_n$. Also show that if symmetric difference of two sets A and B is measurable and B is also measurable then show that A is also measurable. 7+3

6. Prove that E is measurable if and only if given $\varepsilon > 0$, there exist G_s-set $G \supseteq E$ such that $m^*(G - E) = 0$. 10

7. If $< A_n >$ is a countable collection of measurable sets, then prove that $\bigcup_{n=1}^{\infty} A_n$ is also measurable (A'_{ns} may or may not be disjoint). 10

8. A E_1, E_2 are two measurable sets, prove that $E_1 \cup E_2$ is also measurable. Deduce that $E_2 - E_1$ is also measurable. 7+3

UNIT—III

9. State and prove Lusin's theorem. 10

10. Define a measurable function. Let $f : [0, 1] \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} \dfrac{1}{\sqrt{x}}, & x \in (0,1] \\ 0, & x = 0 \end{cases}$$

Let $\alpha \in \mathbb{R}$, find $\{x \in [0, 1] \mid f(x) \leq \alpha\}$ and hence check measurability of function f. 10

11. (a) If f, g are measurable function defined on D (D measurable) show that the set

$$\{x \in D : g(x) < f(x)\}$$

is also measurable. 10

(b) Let f be a function defined on measurable set E. Prove that f is measurable function f and only if $f^{-1}(G)$ is measurable set for each open set G in $\mathbb{R}$. 5+5

12. If $< f_n >$ is a sequence of measurable functions defined on a measurable set E and $\lim_{n\to\infty} f_n(x) = f(x)$ a.e. on E, show that f is measurable on E. 10

UNIT—IV

13. (a) Let $\phi = \sum_{i=1}^{n} a_i \psi_{E_i}$ where E'_{ir} are disjoint measurable sets of finite measure, then prove that

$$\int \phi = \sum_{i=1}^{n} a_i mE_i .$$

(b) Let $f : [0, 2] \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} 2, & \text{if } x \text{ is rational point of } [0,2] \\ 0, & \text{if } x \text{ is irrational point of } [0,2] \end{cases}$$

Check the Riemann integrability and Lebesque integrability of f. 5+5

14. State bounded convergence theorem. Give an example to prove that theorem may not be true for Riemann integration. 10

15. If f, g are integrable functions over E, prove that f + g is integrable over E and $\int_E (f+g) = \int_E f + \int_E g$. 10

Exam. Code : 211002
Subject Code : 4929

M.Sc. Mathematics 2nd Semester

TENSORS & DIFFERENTIAL GEOMETRY

Paper—MATH-562

Time Allowed—Three Hours] [Maximum Marks—100

Note :— Attempt *two* questions from each unit. All questions carry equal marks.

UNIT—I

1. Define Cartesian Tensor of order 3. Prove that the product of two tensors of order m and n is a tensor of order m + n.

2. State and prove quotient law of tensor of order 2.

3. Show that g_{ij} is a covariant symmetric tensor of second rank.

4. Show that the laws of transformation for Christoffel symbols possess the transitive property.

UNIT—II

5. Define osculating plane. Find the equation of osculating plane at any point on the given curve.

6. Show that the necessary and sufficient condition for the curve to be a plane curve is $\left[\bar{r}' \ \bar{r}'' \ \bar{r}'''\right] = 0$.

7. Find the radii of curvature and torsion at any point of the curve $x^2 + y^2 = a^2$, $x^2 - y^2 = az$.

8. Define Circular Helix. Prove that if the curvature and torsion are both constant, then the curve is circular helix.

UNIT—III

9. Find the curvature and torsion of the spherical indicatrix of tangents.

10. Prove that, in order that the principal normal of a curve be binormal of another, the relation $a(K^2 + \tau^2) = K$ must hold where a is constant.

11. Find the envelope of the normal planes to the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, $x^2 + y^2 + z^2 = r^2$.

12. Discuss geometrical interpretation of first fundamental form.

UNIT—IV

13. Prove that at any point of the surface, the sum of the radii of normal curvature in conjugate directions is constant.

14. Show that the differential equation of lines of curvature can be put in the form $\left[\hat{N}, \ d\hat{N}, d\bar{r}\right] = 0$.

15. Find the asymptotic lines of the surface $z = f(x, y)$.

16. Obtain the Mainardi-Codazzi relations in their usual form.

UNIT—V

17. Show that on the right circular cone :

$x = u \cos \phi$, $y = u \sin \phi$, $z = u \cot \alpha$, the geodesics are given by $u = h \sec (\phi \sin \alpha + \beta)$, where h, α, and β are constants.

18. Show that the torsion of the geodesic tangent at any point of a curve on a surface is given by $\tau = (K_b - K_a) \sin \psi \cos \psi$, where ψ is the angle between the tangent and principal direction and K_a and K_b are the principal curvatures of the surface.

19. Prove that $K^2 = K_g^2 + K_n^2$, where K, K_g, K_n denote curvature, geodesic curvature and normal curvature.

20. State and prove Tissot's theorem.

16. (a) Prove that every symmetric function in n variables is a rational function of elementary symmetric functions. 5

(b) Prove that every polynomial over rationals of degree ≤ 4 is solvable by radicals. 5

UNIT—V

17. Let R be a ring with unity and M be an R-module. Then the following are equivalent :

(i) M is simple;

(ii) M is non-zero and is generated by every non-zero element of M;

(iii) $M \cong R/I$, for a maximal left ideal I of R. 10

18. Prove that any two bases of a free module over commutative ring have same number of elements. Is the result true for free module over arbitrary ring ? 10

19. Prove that a finitely generated module over PID can be expressed as direct sum of its torsion submodule and a free module. 10

20. (a) Is $\mathbb{Z}_n$ free over $\mathbb{Z}$? Justify your answer. 5

(b) Give an example of a linearly independent subset of free module that cannot be extended to its basis. 5

Exam. Code : 211002
Subject Code : 4930

M.Sc. Mathematics 2nd Semester

ALGEBRA–II

Paper—Math-563

Time Allowed—3 Hours] [Maximum Marks—100

Note :– The candidates are required to attempt **two** questions from each unit. Each question carries equal marks.

UNIT—I

1. (a) Show that the ring of Gaussian integers is an Euclidean Ring. 5

(b) Prove that F[x], F field, is PID. 5

2. (a) Prove that in an Euclidean ring R, <a> is maximal ideal if and only if a is a prime element of R. 5

(b) State Eisenstein's Criterion and prove that $1 + x + ... + x^4$ is irreducible over $\mathbb{Q}$. 5

3. Prove that a if a ring R is PID then it is UFD. Is the converse true ? Justify. 10

4. (a) State and prove Gauss lemma for primitive polynomials. 7

(b) If a is a rational number such that x–a divides a monic polynomial with integral coefficients then prove that a is an integer. 3

UNIT—II

5. (a) Prove that $\sqrt{5}+\sqrt[3]{3}$ is algebraic over $\mathbb{Q}$ of degree 6. 5

(b) Let $F \subseteq K$ be fields. Prove that the set L of all those elements of K which are algebraic over F forms a subfield of K containing F. 5

6. (a) Find the necessary and sufficient conditions on a, b so that the splitting field of $x^3 + ax + b$ is of degree 3 over $\mathbb{Q}$. 5

(b) Let f(x) be a non-constant polynomial over field F, then prove that there exists an extension E of F in which f(x) has a root. 5

7. (a) Let $F \subseteq K$ be fields. If an element a of K has a minimal polynomial of odd degree over F then prove that $F(a) = F(a^2)$. 5

(b) Prove that the splitting of any polynomial over F is a finite extension of F. 5

8. (a) Prove that K is a finite extension of F if and only if K is generated by a finite number of algebraic elements over F. 5

(b) Let θ be a root of an irreducible polynomial

$x^3 + 9x + 6$. Then find $\frac{1}{1+\theta}$ in $\mathbb{Q}(\theta)$. 5

UNIT—III

9. (a) Prove that it is impossible to trisect angle 60 by using ruler and compass only. 5

(b) Is $\sqrt[3]{2}+1$ constructible ? 5

10. (a) Prove that the characteristic of a finite field F is prime number say p and F contains a subfield isomorphic to Z_p. 5

(b) Construct a field with 16 elements. 5

11. (a) Prove that a field of p^n elements has only one subgroup of order p^m for each m dividing n. 5

(b) Show that all the roots of an irreducible polynomial over finite field are distinct. 5

12. Let K be a field of characteristic p. Prove that K is perfect if and only if every element of K has pth root in K. 10

UNIT—IV

13. Prove that for a Galois extension E/F, F is the fixed of G(E/F) and [E : F] = |G (E/F)|. 10

14. Prove that the Galois group of $x^4 - 2$ over $\mathbb{Q}$ is the group of symmetries of a square. 10

15. Suppose that the Galois group G(E/F) of a polynomial f(x) over F is a solvable group, prove that E is solvable by radicals over F. 10

UNIT—V

XVII. Describe Galerkin's method in detail.

XVIII. Find an approximate solution of the equation

$$\frac{d^2z}{dx^2} + \frac{d^2z}{dy^2} = -1 \text{ in the rectangle } D \begin{cases} -a \le x \le a \\ -b \le y \le b \end{cases} \text{ with}$$

$z = 0$ on the boundary of D.

XIX. Find the extremum of the functional

$$I = \int_{x_1}^{x_2} (y'^2 + z'^2 + 2yz)\, dx$$

with $y(0) = 0$, $z(0) = 0$ and the point (x_2, y_2, z_2) moves over the fixed plane $x = x_2$.

XX. Explain Isoperimetric problem.

Exam. Code : 211002
Subject Code : 4931

M.Sc. (Mathematics) Semester—II

MATH–564 : MECHANICS—II

Time Allowed—3 Hours] [Maximum Marks—100

Note :— Attempt **TEN** questions in all, selecting **TWO** questions from each unit. All questions carry equal marks.

UNIT—I

I. Prove that the rate of change of angular momentum about any axis through a fixed point for a system of particles is equal to the sum of the physical moments of the external forces acting on that system about the line.

II. Two particles of masses m and n are connected by a rigid rod. The velocities of these particles are suddenly changed by applying external impulses. Find the magnitude of the impulsive reaction of the rod on m.

III. A uniform circular disc of mass m and radius r is rotating in its plane with initial angular velocity w, while its centre is at rest. If a point on the rim of the disc is suddenly fixed, then determine the new angular velocity of the disc and the velocity of its centre.

IV. Establish the law of conservation of energy for the compound pendulum.

UNIT—II

V. Describe the motion of a heavy sphere on the interior rough surface of a vertical circular cylinder.

VI. Prove that the rigid body motion about a fixed point is equivalent to the rolling of one cone on another.

VII. A rigid body is free to rotate about its centroid G, the principal moments of inertia at which are 7, 25, 32 respectively and the body is given an angular velocity Ω about a line through G whose direction ratios are (4, 0, 3). Find the components of angular velocity about the principal axes of inertia at G.

VIII. Prove that if $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ is the equation of the momental ellipsoid and $Ax + By + Cz = D$ is the equation of the invariable plane, then $\left(\frac{A^2+B^2+C^2}{D^2}\right)$ is constant.

UNIT—III

IX. Derive kinetic energy as a quadratic function of the generalised velocity components.

X. Describe Lagrange's equations.

XI. Two uniform rods AB, AC each of mass m and length 2a, are smoothly hinged together at A and move on a horizontal plane. The centre of mass, at time t, is at the point (x_1, y_1). Find the kinetic energy of the system.

XII. A light string of length 3ℓ is stretched horizontally between two fixed points. Gravity is neglected. Masses 15 m, 7 m are attached to the points of trisection. The tension in equilibrium is $\lambda m\ell$. The particle of mass 15 m is drawn aside a distance a, the other remaining undisplaced and both are simultaneously released. Find the displacement of the particle of mass 7 m.

UNIT—IV

XIII. If a functional $I[y(x)] = \int_a^b F(x, y(x), y'(x))\, dx$ attains a maximum or minimum on $y = y_0(x)$, where the domain of definition belongs to certain class, then at $y = y_0(x)$, prove that $\delta I = 0$.

XIV. Describe Brachistochrone problem.

XV. What is the distinction between Hamilton's principle and the principle of Least Action ? Explain.

XVI. Test for an extremum of the functional :

$$I[y(x)] = \int_0^1 (xy + y^2 - 2y^2y')\, dx, \quad y(0) = 1,\ y(1) = 2.$$

UNIT—V

17. Solve Fredholm's integral equation by the method of successive substitutions.

18. Find the iterated Kernel for the following :

$$K(x, \eta) = x - \eta,\ a = 0,\ b = 1.$$

19. Solve by method of successive approximation :

$$F(x) = \left(\sin x - \frac{x}{4}\right) + \frac{1}{4}\int_0^{\pi/2} \eta\, x\, F(\eta)\, d\eta .$$

20. Show the solution $F(x) = G(x) + \lambda \int_a^b R(x, \eta; \lambda)\, G(\eta)\, d\eta$ of the non-homogeneous Fredholm's integral equation is unique provided that $D(\lambda) \neq 0$.

Centre No-3 EVENING 22/05/17

Exam. Code : 211002
Subject Code : 4932

M.Sc. Mathematics 2nd Semester

DIFFERENTIAL & INTEGRAL EQUATIONS

Paper—MATH-565

Time Allowed—Three Hours] [Maximum Marks—100

Note :— Attempt ***two*** questions from each unit. All questions carry equal marks.

UNIT—I

1. Show that the general solution of the linear partial differential equation $Pp + Qq = R$ is $F(u, v) = 0$ where F is an arbitrary function and $u(x, y, z) = c_1$ and $v(x, y, z) = c_2$ form a solution of the equations

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}.$$

2. Find the general integral of the equation :

$(x - y)\,p + (y - x - z)q = z$ and the particular solution through the circle $z = 1,\ x^2 + y^2 = 1$.

3. Determine the characteristics of the equation $z = p^2 - q^2$ and find the integral surface which passes through the parabola $uz + x^2 = 0,\ y = 0$.

4. Show how to solve, by Jacobi's method, a partial differential equation of $f\left(x, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial z}\right) = g\left(y, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}\right)$ and illustrate the method by finding a complete integral of the equation $2x^2y\left(\frac{\partial u}{\partial x}\right)^2 \frac{\partial u}{\partial z} = x^2 \frac{\partial u}{\partial y} + 2y\left(\frac{\partial u}{\partial x}\right)^2$.

UNIT—II

5. If $\beta_r D' + r_r$ is a factor of F(D, D') and $\phi_r(\xi)$ is an arbitrary function of the single variable ξ, then if $\beta_r \neq 0$, $u_r = \exp\left(\frac{\gamma_r y}{\beta_r}\right)\phi_r(\beta_r x)$ is a solution of the equation F(D, D')z = 0.

6. Solve :

$$x^2 \frac{\partial^2 z}{\partial x^2} - 4xy \frac{\partial^2 z}{\partial x \partial y} + 4y^2 \frac{\partial^2 z}{\partial y^2} + 6y \frac{\partial z}{\partial y} = x^3 y^4.$$

7. Reduce the equation $(n - 1)^2 r - y^{2n} t = ny^{2n-1} q$ to canonical form and obtain its solution.

8. Solve by Monge's method :

$$2x^2 r - 5xys + 2y^2 t + 2(px + qy) = 0.$$

UNIT—III

9. Solve Laplace equation by the method of separation of variable in Cartesian coordinate system.

10. Solve the differential equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ for the conduction of heat along a rod without radiation, subject to the conditions :

(i) u is not finite for $t \to \infty$

(ii) $\frac{\partial u}{\partial x} = 0$ for x = 0 and n = l

(iii) $u = lx - x^2$ for t = 0 between x = 0 and x = l.

11. Solve wave equation in spherical polar coordinates by the method of separation of variables.

12. Use Poisson's integral formula to show that if the function ψ is harmonic in a circle S and continuous on the closure of S, the value of ψ at the centre of S is equal to the arithmetic mean of its value on the circumference of S.

UNIT—IV

13. Obtain the integral equation corresponding to $\frac{d^2y}{dx^2} + y = \cos x$, y(0) = 0, y'(0) = 0.

14. Solve non-homogeneous Volterra's integral equation of second kind by the method of successive approximation.

15. Find the resolvent Kernel of Volterra's integral equation with the following Kernel K(x, η) = cosh x/cosh η.

16. Solve $F(x) = 1 + x + \lambda \int_0^x (x - \eta) F(\eta)\, dx$.