Exam Code: 211001 **Paper Code: 1226**

Programme: Master of Science (Mathematics) Semester-I

Course Title: Real Analysis-I

Course Code: MMSL – 1331

Time Allowed: 3 Hours **Max. Marks: 80**

Note : Attempt five questions in all selecting atleast one question from each section. Fifth question can be attempted from any section. Each question carries equal 16 (8+8) marks.

Section -A

Q-1 (a)Define the Space R^n,and prove that it is a metric Space under the metric d defined as

$$[d(x,y)]^2 = \sum_{i=1}^{n}(x_i - y_{i)}{}^2 \forall \text{ x,y} \in R^n$$

(b) Prove that the countable union of countable sets is countable.

Q-2 (a) Prove that interior of a set is the largest open set containing the set A

(b) Prove that in a usual metric space ,every closed interval is compact.

Section -B

Q-3 (a) State and prove any two properties of cantor ternary set

(b) Prove that real line is connected if it is an interval. Also give an example to show that union of two open sets may not be connected

Q-4 (a) Prove that the sets of all subsequential limits of a sequence form a closed set in a metric space

(b) Prove that every convergent sequence is a Cauchy in a metric space but converse is not true

Section -C

Q-5 (a) State and Prove Baire's Catagory theorem

(b) Define fixed point of a mapping . Give an example for the same. Also define dense set, nowhere dense set , first category set and second category set

Q- 6(a) Define uniform continuity in a metric space and establish a relation between continuity and uniform continuity

(b) if f is mapping from X to Y. Then f is continuous iff image of the closure of A is contained in closure of the image of A

Section-D

Q-7 (a) Give an example of a function which is bounded but not R-integral and also a function which has finite number of discontinuities but RS-Integrable ; justify your answer

(b) State and Prove Theorem connected to R- integral and RS-Integral

Q-8 (a) State and prove Fundamental theorem of calculus

(b) Evaluate $\int_0^3 e^x \, d(x - [x])$

Exam Code: 221001 **Paper Code: 1227**
(20)

Programme: Master of Science (Mathematics)
Semester-I

Course Title: Complex Analysis

Course Code: MMSL-1332

Time Allowed: 3 Hours **Max Marks: 80**

Candidates are required to attempt five questions in all, selecting at least one question from each section. The fifth question may be attempted from any of these four sections. Each question carries 16 marks.

Section A

1. (a) Find necessary and sufficient Condition for a given function f (z) to be analytic in given region R.

   (b) Show that f (z) = $|z|$is not differentiable anywhere.

   (c) Evaluate $\int_C \frac{dz}{z}$ where C denotes the square described in the positive sense with sides parallel to the axis and of length 2a and having its center at the origin.

2. (a) State and prove Cauchy integral theorem.

(b) Evaluate $\int_C \frac{e^{2z}}{(z-1)(z-2)} dz$, where C is the circle $|z| = 3$

Section B

3. (a) State and prove converse of Cauchy's integral theorem.

(b) The transformation $w = \frac{1}{z}$ maps a circle in Z- plane to a circle in W-plane to a straight line of the circle in Z-plane passes through the origin.

4. (a) Let f(z) be an analytic function of z in a region D of the Z-plane and let $f'(z) \neq 0$ inside D. Then the mapping $w = f(z)$ is conformal at the points of D

(b) Find the bilinear transformation which maps $z = 1, i, -1$ respectively onto $w = i, 0, -i$

Section C

5. (a) Expand f (z) $= \frac{1}{(z+3)(z+3)}$ in a Laurent series valid for

   (i) $1 < |z| < 3$

   (ii) $|z| > 3$

   (b) State and prove Rouche's theorem.

6. (a) Show that an analytic function can come arbitrary close to any complex number in a neighbourhood of an essential singularity.

   (b) What kind of singularity have the following function?

   (i) $tan \frac{1}{z}$ at $z = 0$

   (ii) $z \, cosec \, z \; at \; z = \infty$

Section D

7. (a) State and prove Cauchy's residue theorem.

(b) Show by contontour integration, prove that $\int_0^{\infty} \frac{dx}{1+x^2} = \frac{\pi}{2}$

8. (a) Find the poles and residues of the following function

$$\frac{1}{Z^m(1-z)^n}$$

(b) Show that $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta} = \frac{2\pi}{\sqrt{3}}$

Exam Code: 221001 **Paper Code: 1228**
(20)

Programme: Master of Science (Mathematics)
Semester-I

Course Title: Algebra-I

Course Code: MMSL-1333

Time Allowed: 3 Hours **Max Marks: 80**

Note: Attempt five question in all, selecting at least one question from each section. The fifth question can be done from any section. Each question carries 16 marks.

Section A

1. (a) Prove that a finite semi group in which cancellation laws hold is a group. (5)

(b) If in a group G, $xy^2 = y^3x$ and $xy^2 = x^3 y$, then show that $x = y = e$, Where e is the identity of G. (5)

(c) Show that there does not exist any group G, such that $|\frac{G}{Z(G)}| = 37$. (6)

2. (a) Prove that intersection of two subgroups of finite index, of a group is of finite index. (6)

(b) Give an example of a non-abelian group in which all subgroups are normal. (4)

(c) Prove that every subgroup of a cyclic group is cyclic. (6)

Section-B

3. (a) State and prove third theorem on Homomorphism. (10)

(b) Prove that every group of order 6 is either cyclic or isomorphic to S_3. (6)

4. (a) If a permutation is a product of even number of transpositions, then it cannot be product of odd number of transpositions. (6)

(b) Show that the only proper non-trivial normal subgroup of S_n is A_n, $n \geq 5$. (5)

(c) Prove that Klein's four group is the direct product of its subgroups. (5)

Section-C

5. (a) State and prove Sylow's first theorem. (10)

   (b) Show that a group of order 108 is simple. (6)

6. (a) Prove that direct product of two solvable groups is solvable. (6)

   (b) State and prove Jordan Holder Theorem. (10)

Section-D

7. (a) Prove or disprove that there exists an integral domain with six elements. (6)

   (b) Prove that every field is a simple ring. (5)

   (c) Let R be a commutative ring with unity in which each ideal is prime. Prove that R is a field. (5)

8. (a) Prove that a ring with unity and having no proper right ideals is a division ring. (6)

(b) If R is a ring with unity, then each maximal ideal is prime. What about converse? (10)

Exam Code: 221001
(20)

Paper Code: 1229

Programme: Master of Science (Mathematics)
Semester-I

Course Title: Mechanics-I

Course Code: MMSL-1334

Time Allowed: 3 Hours

Max Marks: 80

Note: Attempt five questions in all, selecting at least one question from each section. The fifth question can be attempted from any section. Each question carries 16 marks.

Section — A

1. (a) Obtain the radial and transverse components of velocity and acceleration of a particle which describe the plane curve $r = f(\theta)$.

(b) If the curve is the equiangular spiral $r = a \exp(\theta \cot \alpha)$ and if the radius vector to the particle has a constant angular velocity, show that the resultant acceleration of the particle makes an angle 2α with the radius vector and its magnitude v^2/r, where v is the speed of the particle. (8,8)

2. An aircraft pursues a straight course with constant velocity V and is being chased by a guided missile moving with speed 2V and fitted with a homing device to ensure that its motion is always directed at the target. Initially the missile is at right angles to the course of the aircraft and distant R from it. Find the polar equation of the missile's pursuit curve relative to the target, taking the course of the target as the initial line $\theta = 0$ and find the time taken by it to strike the target. (16)

Section- B

3. Discuss constrained particle motion with an example. (16)

4. Discuss cycloid and its dynamical properties. (16)

Section -C

5. (a)Derive an expression for the differential equation of a particle moving in a central orbit in pedal coordinates.

   (b) A particle is placed at the highest point of a smooth vertical circle of radius a and is allowed to slid down starting with a negligibie velocity. Prove that it will leave the circle after describing vertically a distance equal to one third of the radius of the circle.

   (8,8)

6. (a) A particle is projected upward with a velocity V in a medium whose resistance varies as the square of the velocity. Discuss the motion.

   (b) If $P = \mu(u^2 - au^3)$, where $a > 0$ and a particle is projected from an apse at a distance a from the centre of force with velocity $(\mu c/a^2)^{1/2}$, where $a > c$, prove that the other apsoidal distance of the orbit is $a(a+c)/(a-c)$ and find the apsoidal angle.

   (8,8)

Section — D

7. Define Principal Axes. Show that at each point of a rigid body there are three principal axes mutually at right angles to each other such that the product of inertia about them taken two at a time is zero. Hence deduce the value of angular momentum of the rigid body referred to these axes. (16)

8. (a) State and prove theorem of parallel axes for moments of inertia.

   (b) A square of side 2a has particles of masses m, 2m, 3m, 4m at its vertices. Find the principal moments of inertia at the centre of the square and also the directions of the principal axes. (8,8)

Exam Code: 221001 **Paper Code: 1230**
(20)

Programme: Master of Science (Mathematics) Semester-I

Course Title: Differential Equations

Course Code: MMSL-1335

Time Allowed: 3 Hours **Max Marks: 80**

Note: Attempt five questions, selecting at least one question from each section. The fifth question may be attempted from any section. Each question carries equal marks.

Section-A

1(a) Apply Picard's method of Successive approximations to solve the initial value problem up to third approximation: $\frac{dy}{dx} = x + y^2$ given that $y = 0$ when $x = 0$

(b) Solve the equation: $\frac{dx}{y(x+y)+az} = \frac{dy}{x(x+y)-az} = \frac{dz}{z(x+y)}$ (8,8)

2(a) State and Prove Sturm's Comparison Theorem

(b) Prove that eigen functions corresponding to the different eigen values of a Sturm Liouville problem are orthogonal. (8,8)

Section-B

3(a) State Convolution Theorem and hence evaluate $L^{-1}\left(\frac{s+2}{(s^2+4s+5)^2}\right)$

(b) Evaluate L { $t^2 e^{-t} \sin t$) (10,6)

4(a) Using Laplace transform, solve the differential equation

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 3te^{-t}; y(0) = 4, y'(0) = 2$$

(b) Apply Heaviside expansion Theorem to obtain $L^{-1}\left(\frac{2s^2-6s+5}{s^3-6s^2+11s-6}\right)$ (8,8)

Section-C

5(a) Find the complex Fourier transform of $F(x) = \begin{cases} 1 - x^2, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$ and hence evaluate

$\int_0^\infty \left(\frac{x\cos x - \sin x}{x^3}\right) \cos\frac{x}{2}\, dx$

(b) If $f(s)$ is the Fourier transform of $F(x)$, then $\left(\frac{1}{|a|}\right) f\left(\frac{s}{a}\right)$ is the Fourier Transform of

$F(ax); a \neq 0$ (10,6)

6(a) Find Fourier Sine and Cosine Transform of $f(x) = x^2; 0 < x < \pi$

(b) Express the function $f(x) = \begin{cases} 1, & |x| \le 2 \\ 0, & |x| > 2 \end{cases}$ as a Fourier integral. (10,6)

Section-D

7(a)Show that $J_n(x) = \frac{1}{\pi}\int_0^\pi \cos(n\varphi - x\sin\varphi)d\varphi$, *where n is a positive integer*

(b) Prove that $P_n(x) = \frac{1}{2^n\, n!}\frac{d^n}{dx^n}(x^2 - 1)^n$. (8,8)

8(a) Prove that $\int_0^\infty e^{-x} L_n(x) L_m(x)\, dx = \begin{cases} 0, & m \neq n \\ 1, & m = n \end{cases}$

(b) Show that $H_{2n}(0) = (-1)^n \frac{(2n)!}{n!}$ (8,8)