Exam Code: 221002 **Paper Code: 2225** **(20)**

Programme: Master of Science (Mathematics) Semester-II

Course Title: Real Analysis-II

Course Code: MMSL-2331

Time Allowed: 3 Hours **Max Marks: 80**

1) **Paper consists of Eight questions of equal marks (16 mark each)**
2) **Attempt Five questions in all by selecting atleast One question from each of Four section. FIFTH question may be attempted from any section.**

SECTION-A

1. (a) Let $\{f_n\}$ be a sequence of real valued functions defined on the closed and bounded interval $[a,b]$ and let $f_n \in R[a,b]$, for $n = 1,2,3,\ldots$. If f_n converges uniformily to the function f on $[a,b]$, then prove that $f \in R[a,b]$ and

$$\int_a^b f(x)dx = \lim_{n\to\infty} \int_a^b f_n(x)dx$$

(10)

(b) Examine for term by term integration the series, the sum of whose first n terms is

$$n^2x(1-x)^n \qquad (0 \le x \le 1)$$

(6)

2. State and prove Stone-Weierstrass Theorem. (16)

SECTION-B

3. (a) Prove that the outer measure of an interval is its length. (10)

(b) Find the length of the set $\bigcup_{k=1}^{\infty} \left\{x : \frac{1}{k+1} \le x < \frac{1}{k}\right\}$. (6)

4. (a) Let $\{E_i\}$ be an infinite increasing sequence of measurable sets, that is, a sequence with $E_{i+1} \supset E_i$ for each $i \in \mathbb{N}$. Then prove that

$$m\left(\bigcup_{i=1}^{\infty} E_i\right) = \lim_{n\to\infty} m(E_n)$$

(8)

(b) Let f be a function defined on a measurable set E. Then prove that f is measurable if and only if, for any open set G in $\mathbb{R}$, the inverse image $f^{-1}(G)$ is a measurable set. (8)

SECTION-C

5. A bounded function f defined on a measurable set E of finite measure is Lebesgue integrable if and only if f is measurable. (16)

6. (a) Let f be a bounded fucntion defined on $[a,b]$, If f is Reimann integrable over $[a,b]$, then it is Lebesgue integrable and

$$\Re\int_a^b f(x)dx = \int_a^b f(x)dx$$

. (8)

(b) **(Monotone Convergence Theorem)** Let $\{f_n\}$ be an increasing sequence of non-negative measurable functions, and let $f = \lim_{n\to\infty} f_n$. Then prove that

$$\int f = \lim_{n\to\infty}\int f_n$$

. (8)

SECTION-D

7. (a) If f is any function on an interval I, then prove that $\overline{D}f$ and $\underline{D}f$ are measurable functions. (8)

(b) Prove that the union of any collection of intervals is a measurable set. (8)

8. Let f be an increasing real-valued function defined on $[a,b]$. Then prove that f is differentiable a.e.(*almost everywhere*) and the derivative f' is measurable. Furthermore,

$$\int_a^b f'(x)dx \leq f(b) - f(a)$$

. (16)

Exam Code: 221002 **Paper Code: 2226**

(20)

Programme: Master of Science (Mathematics)
Semester-II

Course Title: Tensors and Differential Geometry

Course Code: MMSL-2332

Time Allowed: 3 Hours **Max Marks: 80**

Attempt five questions, selecting at least one question from each section. The fifth question may be attempted from any section. Each question carries 16 marks.

Section-A

1. (a) Show that inner product of the tensors A_r^p and B_t^{qs} is a tensor of rank three (4)

   (b) If a_{ij} is a second rank covariant symmetric tensor and $|a_{ij}|$ = a then show that $\sqrt{a}$ is a scalar density. (4)

   (c) Find the metric of a Euclidean space referred to cylindrical co-ordinates. (8)

2. (a) Show that the covariant derivative of a contra variant vector is a mixed tensor of rank two. (8)

   (b) Show that g_{ij} is a second rank covariant symmetric tensor. (8)

Section-B

3. (a) Find the radii of curvature and torsion for the curve $x = 3u, y = 3u^2, z = 2u^3$. (8)

   (b) Prove that for all helices curvature bears a constant ratio with torsion. (8)

4. (a) Show that a necessary and sufficient condition for a curve lies on a sphere is that $\frac{f}{\sigma} + \frac{d}{ds}\left(\frac{p^1}{z}\right)$=0 at every point on the curve. (8)

   (b) Obtain the curvature and torsion of spherical indicatrix of the tangent. (8)

Section-C

5. (a) Prove that necessary and sufficient condition for a surface to be developable surface is that its Gaussian curvature should be zero. (8)

   (b) State and prove Beltranil and Enneper theorem.(8)

6. (a) Obtain the Manardi-codazzi equations in their usual form. (8)

   (b) Find the differential equation of lines of curvature of the helicoid $x = u \cos v \, , y = \, u \sin v \, and$ $z = \, f(u) + cv.$ (8)

Section-D

7. (a) State and prove necessary and sufficient condition for a curve on a surface to be geodesic. (8)

   (b) Prove that two geodesic at right angles have their torsions equal in magnitude but opposite in sign. (5)

   (c) Prove that the curvature of a geodesic relative to itself is zero. (3)

8. (a) State and prove Gauss-Bonnet theorem. (8)

   (b) State and prove Joachimsthal theorem. (8)

Exam Code: 221002 **Paper Code: 2227**
(20)

Programme: Master of Science (Mathematics)
Semester-II

Course Title: Algebra-II

Course Code: MMSL-2333

Time Allowed: 3 Hours **Max Marks: 80**

Attempt five questions selecting at least one question from each section. The fifth question may be attempted from any section. Each question carries 16 marks.

Section-A

1. (a) Let R be commutative ring with unity then R$[x]$ is P.I.D iff R is a field. 12

   (b) Is $\frac{\oiint [x]}{<x^2-5x=6>}$ is a field? Why? Here $\oiint$ is a field of rationals. 04

2. (a) Every Euclidean domain is Principal Ideal domain.06

   (b) Let D be a U.F.D. and $f(x) \in D[x]$ be anirreducible element of D$[x]$ then either $f(x)$ is irreducible element of D or $f(x)$irreducible primitive polynomial over F, where F is field of quotient of D. 10

Section-B

3. (a) If E is finite extension of finite field F then E is simple extension of F. 08

(b) Show by an example that finitely generated field extension may not be finite extension. 04

(c) Let K be field extension of F and $a \in k$ is algebraic over F of an odd degree then F (a) = F (a^2) 04

4. (a) If F is finite field of characteristic þ, show that each element 'a' of F has a unique pth root in F. 05

(b) Let α be a root of $x^þ - x - 1$ over a field F of characteristic p. Show that F(α) is a separable extension of F. 04

(c) Let K be a field extension of F, $a \in k$ be algebraic over F then F[a] is field. 07

Section-C

5. (a) Prove or disprove if F⊆E⊆K be chain of fields such that E is normal extension of F and K is normal extension of E then K is normal extension of F. 06

(b) Show that Galois group of $x^4 - 2 \in \mathbb{Q}[x]$ is the octic group. 10

6. (a) If $f(x) \in F[x]$ has r distinct roots in its splitting field E over F then the Galois group G(E/F) of $f(x)$ is a subgroup of symmetric group S_r. 10

(b) Show that the polynomial $x^7 - 10x^5 + 15x + 5$ is not solvable by radicals over $\mathbb{Q}$, field of rationals.

06

Section-D

7. (a) Prove that over a commutative ring, any two basis of a finitely generated free module have same number of elements. 10

   (b) Let M be cyclic R-module that is M=Rm for same m∈M then $M \cong \frac{R}{I}$ for some left ideal I of R. 06

8. (a) Let R be a ring with unity. Prove that in an R-module R a left ideal A is direct summand of R iff A=Re for same idempotent e of R. 6

   (b) Prove or disprove that every submodule of an R-module is direct summand. 05

   (c) Show by an example that submodule of finitely generated module may not be finitely generated. 05

Exam Code: 221002 **Paper Code: 2228**
(20)

Programme: Master of Science (Mathematics) Semester-II

Course Title: Mechanics-II

Course Code: MMSL-2334

Time Allowed: 3 Hours **Max Marks: 80**

Attempt a total of five questions selecting at least one question from each section. The fifth question may be attempted from any section. Each question carries 16 marks.

Section A

1. (a) State and prove the principle of conservation of angular momentum. [8]
   (b) Establish that the linear momentum is constant for a system of particles having no resultant force. [4]
   (c) Derive an expression for $K.E.$ of a rigid body moving in two dimensions. [4]

2. (a) A uniform rod of mass M and length $2a$ lies at rest on a smooth horizontal table. An impulse J is applied at A in the plane of the table and perpendicular to the rod. Determine the velocity of the centroid and the angular velocity of the rod. [4]
   (b) Show that the rate of change of vector angular momentum of a system of particles moving generally in space is equal to the moments of the external forces acting on the system.(External and internal forces). [6]
   (c) Two particles of masses m_1 and m_2 at A and B are connected by a rigid rod AB lying on a smooth horizontal table. If an impulse I is applied at A in the plane of the table and perpendicular to AB. Find the initial velocities of A and B. [6]

Section B

3. (a) Find the kinetic energy of a rigid body rotating about a fixed point with velocity $\vec{\varpi}$. [8]
   (b) Prove that a rigid body motion about a fixed point under no force is equivalent to rolling of one cone on another. [8]

4. (a) Show that the product of inertia with respect to principal axes vanish. [10]
   (b) Show that vector angular momentum about a fixed point of a particle moving under no forces is constant. [6]

Section C

5. (a) Two uniform rods AB, AC each of mass m and length $2a$, are smoothly hinged together at A and move on a horizontal plane. At time t the mass-centre of the rods is at the point (ξ, η) referred to fixed perpendicular axes Ox, Oy in the plane and the rods make angle $\theta \pm \phi$ with Ox. Prove that the kinetic energy of the system is $m[\dot{\xi}^2 + \dot{\eta}^2 + \left(\frac{1}{3} + \sin^2\phi\right) a^2\dot{\theta}^2 + \left(\frac{1}{3} + \cos^2\phi\right) a^2\dot{\phi}^2]$. [8]
   (b) Derive Lagrange's equations for Impulsive Forces. [8]

6. (a) Determine Lagrange's equations of the motion of a planet of mass m orbiting round the sun under inverse square law of attraction. [6]

(b) A horizontal circular wire has radius R, centre C and is free to rotate about a vertical axis through a point O in its plane distant d from C. The wire carries a smooth particle P and $\angle OCP = \theta$ at time t. If ω is the angular velocity of the wire, show that $R\ddot{\theta}+\dot{\omega}(R-d\cos\theta) = d\omega^2 \sin\theta$. [6]

(c) Define

(i) Generalised coordinates of a dynamical system

(ii) Impulsive virtual work function.

(iii) Generalised forces.

(iv) Holonomic System. [4]

Section D

7. (a) Find the extremals of the functional $\int_0^1 (xy + y^2 - 2y^2y')dx$; $y(0) = 1\, and\, y(1) = 2$ [3]

(b) Show that the geodesics on a sphere of radius a are its great circles. [8]

(c) Find approximately the smallest eigen value λ of $y'' + \lambda y = 0$; $y(0) = y(1) = 0$ [5]

8. (a) Find the equation of motion of one dimensional harmonic oscillator using hamilton's principle. [8]

(b) Distinguish between Hamilton's Principle and the Principle of Least Squares. [4]

(c) Solve the boundary value problem $y'' - y + x = 0\,(0 \leq x \leq 1)$, $y(0) = y(1) = 0$ by Rayleigh Ritz method. [4]

Exam Code: 221002 **Paper Code: 2229**
(20)

Programme: Master of Science (Mathematics) Semester-II

Course Title: Differential and Integral Equations

Course Code: MMSL-2335

Time Allowed: 3 Hours **Max Marks: 80**

Note: Attempt five questions in all, selecting at least one question from each section. The fifth question may be attempted from any section. Each question carries 16 marks.

Section-A

1(a) Find the general integral of the equation $(x-y)p+(y-x-z)q=z$ and the particular solution through the circle $z=1, x^2+y^2=1$

(b) Show that the equations $xp-yq=x, x^2p+q=xz$ are compatible and find their solution.

[8,8]

2(a) Show how to solve, by JACOBI METHOD, a partial differential equation of the type $f(x, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial z})=g(y, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z})$ and illustrate the method by finding a complete integral of the equation $2x^2y\left(\frac{\partial u}{\partial x}\right)^2\frac{\partial u}{\partial z}=x^2\frac{\partial u}{\partial y}+2y\left(\frac{\partial u}{\partial x}\right)^2$

(b) Solve: $r+s-6t=y\cos x$ [10,6]

Section-B

3 (a) Reduce $x\frac{\partial^2 z}{\partial x^2}+2\frac{\partial^2 z}{\partial x \partial y}+\frac{\partial^2 z}{\partial y^2}=0$ to canonical form and hence solve it.

(b) Obtain the solution of $\frac{\partial^2 z}{\partial x \partial y}=\frac{1}{x+y}$ such that $z=0, p=\frac{2y}{x+y}$ $on\ y=x$.

[8,8]

4(a) Find the solution of Laplace's equation in two dimensions by the method of Separation of Variables.

(b)Solve the Wave equation $r=t$ by MONGE'S METHOD. [8,8]

Section-C

5. Explain the method of successive substitutions for the solution of Volterra Integral Equation of second kind. [16]

6(a) Form an Integral equation corresponding to the following differential equation with the given initial conditions $y''+y=cosx, \quad y(0)=0, y'(0)=1$

(b) With the help of Resolvent kernel, find the solution of

$$F(x)=e^{x^2}+\int_0^x e^{x^2-\eta^2}F(\eta)d\eta$$ [8,8]

Section-D

7(a) Solve the Fredholm equation

$$u(x)=e^x-\frac{e-1}{2}+\frac{1}{2}\int_0^1 u(t)dt$$

(b) Compute $D(\lambda)$ for the Integral equation

$$y(x)=f(x)+\lambda\int_0^1(x+t)\,y(t)dt$$ [8,8]

8. Show that the solution $F(x)=G(x)+\lambda\int_a^b R(x,\eta;\lambda)G(\eta)d\eta$ of the non-homogeneous Fredholm's integral equation of second kind is unique provided $D(\lambda)\neq 0$ [16]

Exam Code: 226502 **Paper Code: 2169**
(20)

Programme: Master of Arts (Punjabi)
SEMESTER-II

Course Title: Bhagat Bani

Course Code: MPBL-2421

Time Allowed: 3 Hours **Max Marks: 80**

ਵਿਦਿਆਰਥੀ ਨੇ ਕੁੱਲ ਪੰਜ ਪ੍ਰਸ਼ਨ ਕਰਨੇ ਹਨ। ਹਰ ਸੈਕਸ਼ਨ ਵਿਚੋਂ ਇਕ ਪ੍ਰਸ਼ਨ ਕਰਨਾ ਲਾਜ਼ਮੀ ਹੈ। ਪੰਜਵਾਂ ਪ੍ਰਸ਼ਨ ਕਿਸੇ ਵੀ ਸੈਕਸ਼ਨ ਵਿਚੋਂ ਕੀਤਾ ਜਾ ਸਕਦਾ ਹੈ। ਹਰੇਕ ਪ੍ਰਸ਼ਨ ਦੇ 16 ਅੰਕ ਹਨ।

ਸੈਕਸ਼ਨ-1

1. ਭਗਤੀ ਸਾਹਿਤ ਦੇ ਆਰੰਭ ਤੇ ਵਿਕਾਸ ਉਪਰ ਨੋਟ ਲਿਖੋ। 16
2. ਭਗਤੀ ਸਾਹਿਤ ਦੀ ਭਾਰਤੀ ਸਭਿਆਚਾਰ ਨੂੰ ਦੇਣ ਬਾਰੇ ਚਰਚਾ ਕਰੋ। 16

ਸੈਕਸ਼ਨ-2

3. ਭਗਤ ਨਾਮਦੇਵ ਦੀ ਬਾਣੀ ਵਿਚਲੀਆਂ ਕਾਵਿ-ਜੁਗਤਾਂ ਨੂੰ ਉਦਾਹਰਣਾਂ ਸਹਿਤ ਸਪੱਸ਼ਟ ਕਰੋ। 16
4. ਨਾਮਦੇਵ ਬਾਣੀ ਦਾ ਸਮਾਜ ਸਭਿਆਚਾਰਕ ਸੰਦਰਭ ਉਲੀਕੋ। 16