Exam Code: 211003 Paper Code: 3223
(20)

Programme: Master of Science (Mathematics)
Semester-III

Course Title: Functional Analysis-I

Course Code: MMSL-3331

Time Allowed: 3 Hours Max Marks: 80

Note: Attempt five questions in all selecting at least one question from each section. Fifth question may be attempted from any Section. Each question carries equal marks.

Section -A

Q-1 (a) Prove that the real linear space C[0,1]. Of all continuously differentiable functions defined on [0,1] equipped with norm given by $\|x\| = \sup\{|x(t)|: t \in [0,1]\}\ \forall\, x \in C[0,1]$ is an incomplete normed space.

(b) Prove that space of all convergent sequences 'c' is subspace of l^{∞} and is closed [8,8]

Q- 2 a) State and Prove Riesz -Fischer Theorem

b) Prove that the norms $\|T\| = \text{Sup}\{\|T(x)\|: \|x\|=1\}$ and $\|T\| = \inf\{K: K \geq 0 \text{ and } \|T(x)\| \leq K\|x\|\ \forall x \in X\}$ are equivalent for a linear transformation T [8,8]

Section B

Q-3 (a) Prove that dual space of l_1 is l_{∞}

(b) Prove that closed subspace of a reflexive Banach Space is reflexive [8,8]

Q-4 (a) If N is a normed linear Space and x_0 is a non zero vector in N, then there exists a functional f_0 in N^* such that $f_0(x_0) = \|x_0\|$ and $\|f_0\| = 1$

(b) State and Prove Hahn Banach Lemma [8,8]

Section -C

Q-5 (a) State and Prove Banach Steinhaus Theorem

(b) Give an example of continuous mapping which is not open mapping [8,8]

Q-6 Prove that a one to one continuous linear transformation of one Banach space onto another is a homeomorphism [16]

Section-D

Q-7 (a) Prove that a closed convex subset C of a Hilbert Space H contains a unique vector of smallest norm and show that the parallelogram law is not true in the Banach Space l_1^n

(b) If H is a Hilbert Space, then H is reflexive. [8,8]

Q-8 a) Show that the mapping $\varphi: H \to H^*$ defined by $\varphi(y) = f_y$ where $f_y(x) = \langle x, y \rangle\ \forall\, x \in H$ is one to one, onto, additive but not linear and is isometry.

(b) State and prove Bessel's Generalized Inequality [8,8]

Exam Code: 211003 **Paper Code: 3224**
(20)

Programme: Master of Science (Mathematics) Semester-III

Course Title: Topology-I

Course Code: MMSL-3332

Time Allowed: 3 Hours **Max Marks: 80**

Attempt five questions in all, selecting at least one question from each section. The fifth question may be attempted from any Section. Each question carries equal marks.

Section-A

1. (a) Give characterization of topology in terms of Kuratowski function. (8)

   (b) Every metric space is Second countable. Prove or disprove. (8)

2. (a) Prove that First axiom is hereditary property. (8)

   (b) Let (X, τ) be a topological space, then a family $\beta \subseteq \tau$ is a base for τ iff every τ —open set can be expressed as union of members of β (8)

Section-B

3. (a) Prove that union of a collection of connected subspaces of X that have a point in common is connected. (8)

   (b) Give an example of a topological space which is connected but not locally connected. (8)

4. (a) Define Homeomorphism. If $f: R \rightarrow R$ be defined as $f(x) = x + 10$, then prove that f is open. Is f a homeomorphism? (8)

   (b) Let $f: X \rightarrow Y$ be a mapping from first countable space X into a space Y. Then f is continuous at $x_0 \in X$ iff f is sequentially continuous at x_0. (8)

Section-C

5. (a) Prove that product of two regular spaces is a regular space. (8)

   (b) Prove that every T_3 space is T_2- space but converse need not be true. (8)

6. (a) Prove that the property of being a T_4 space is hereditary property. (8)

   (b) State and Prove Tietze-Extension theorem. (8)

Section-D

7. (a) Prove that product space of arbitrary many Hausdorff spaces is Hausdorff. (8)

   (b) Projection mappings are closed mappings. Prove or Disprove. (8)

8. (a) If the quotient space $\frac{X}{R}$ is Hausdorff then prove that R is closed in product space XXX. (8)

   (b) Define Quotient map. Give an example of a quotient map which is not closed. (8)

Exam Code: 211003

Paper Code: 3225

Programme: Master of Science Mathematics,
Semester-III
Course Title - DISCRETE MATHEMATICS-I,
Course Code:MMSL-3333(Opt-I)

Time Allowed: 3 hours,

Maximum Marks: 80

Attempt FIVE questions in all by selecting atleast ONE question from each section. FIFTH question may be attempted from any Section. Each question carries equal marks.

SECTION-A

1. (a) Let $A = \{a_1, a_2, a_3, \ldots, a_n\}$ be a set consisting of n elements. Then prove that total number of anti-symmetric relations in $A \times A$ are $(2^n)(3^{(n^2-n)/2})$ on set A. (8)

   (b) Let A be a finite set and (A, R) is a poset, then prove that A has both a maximal and a minimal element. (4)

   (c) Let $A = \{1,2,3,4,5\} \times \{1,2,3,4,5\}$ and define relation R on A by $(x_1, y_1)R(x_2, y_2)$ if $x_1 + y_1 = x_2 + y_2$ for $x_i, y_i \in \{1,2,3,4,5\}$. Verify that R is an equivalence relation on A and also determine the partition of A induced by R. (4)

2. (a) Let $ABCD$ be a square with $AB = 1$. Show that if we select five points in the interior of this square, there are at least two whose distance apart is less than $\frac{1}{\sqrt{2}}$. (8)

   (b) Let $p_1, p_2, \ldots, p_n \in \mathbb{Z}^+$. Prove that if $p_1 + p_2 + \cdots + p_n - n + 1$ pigeons occupy n pigeonholes, then either the first pigeonhole has p_1 or more pigeons roosting in it, or the second pigeonhole has p_2 or more pigeons roosting in it ,..., or the n^{th} pigeonhole has p_n or more pigeons roosting in it. (8)

SECTION-B

3. (a) Negate and simplify the compound statement $p \wedge (q \vee r) \wedge (\neg p \vee \neg q \vee r)$. (8)

   (b) Using laws of logic prove that $(p \vee q) \wedge \neg(\neg p \wedge q) \Leftrightarrow p$. (8)

4. (a) Use truth tables to verify the logical equivalences $[p \to (q \vee r)] \Leftrightarrow [\neg r \to (p \to q)]$. (8)

   (b) Express the statements in the argument using quantifiers

   "All lions are fierce"
   "Some lions do not drink coffee"
   "Some fierce creatures do not drink coffee"

   (8)

SECTION-C

5. Let S, T, U, and V be sets and let $X \subseteq S \times T$, $Y \subseteq T \times U$, and $Z \subseteq U \times V$ be subsets. Define

$$X * Y := \{(s, u) \in S \times U | \exists t \in T : (s, t) \in X \text{ and } (t, u) \in Y\} \subseteq S \times U$$

(a) Prove that $(X * Y) * Z = X * (Y * Z)$

(b) Let S be a set. Show that $(P(S \times S), *)$ is a monoid. Is it commutative? Here $P(.)$ denotes the power set.

(c) Find the invertible elements in the monoid of Question 6, part (b).

(16)

6. (a) Let G be a semigroup which has a left identity element e such that every element of G has a left inverse with respect to e, i.e., for every $x \in G$ there exists an element $y \in G$ with $yx = e$. Show that e is an identity element and that each element of G is invertible. (8)

(b) If $(S_1, +)$ and $(S_2, .)$ are semigroups, then prove that $(S_1 \times S_2, *)$ is also a semigroup, where operation $'*'$ defined as $(s_1', s_2') * (s_1'', s_2'') = (s_1' + s_1'', s_2'.s_2'')$. (8)

SECTION-D

7. (a) Solve the following recurrence relation by the method of generating functions

$$a_{n+2} - 3a_{n+1} + 2a_n = 0, \quad n \geq 0, \quad a_0 = 1, \ a_1 = 6$$

(8)

(b) Find the generating function for the sequence $a_0, a_1, a_2, \ldots,$ where

$$a_n = \sum_{i=0}^{n} \frac{1}{i!}, \quad n \in \mathbb{N}$$

(8)

8. (a) Let n lines be drawn in the plane such that each line intersects every other line but no three lines are ever coincident. For $n \geq 0$, let a_n count the number of regions into which the plane is separated by the n lines. Find and solve a recurrence relation for a_n. (8)

(b) For the situation defined in above Question 8 part (a), let b_n count the number of infinite regions that result. Find and solve a reccurence relation for b_n. (8)

For Reappear Candidates (2022-23)

Exam Code: 211003 **Paper Code: 9307**

Programme: Master of Science (Mathematics) Semester-III
Courser Title: Integral Transforms
Course Code: MMSL-3333 (Opt-II)

Time Allowed: 3 Hours **Max Marks: 80**

Instructions: Attempt five questions in all selecting atleast one question from each section and fifth question can be attempted from any section. Each question carries equal (16) marks.

Section-A

Q1. (a) When F(x) = Sin nx ; n is an integer . Show that

f_c (m) = 0 if n-m is even

f_c (m) = $\frac{2n}{n^2-m^2}$ if n-m is odd , where m=1,2,3........... (8)

(b) State and prove Inversion formula for Finite Fourier Cosine transform. (8)

Q2. (a) Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, 0 < X < 4 , t > 0

Subject to condition u(0,t)=0, u(4,t)=0 and u(x,0)=2x. Also give a physical interpretation of the problem.

(10)

(b) Find F(x) if its finite Sine transform is given by

$\overline{F_S(p)} = \frac{2\pi(-1)^{p-1}}{p^3}$; p= 1,2,3..... where 0 < x < π.

(6)

Section-B

Q3. (a) Find the Bounded solution of

$\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial x^2}$, y(x,0) = 3 Sin 2πx, y(0,t) = 0, y(1,t) = 0 , 0 < x < 1 , t > 0.

(10)

(b) Evaluate L^{-1} { $\frac{p^2}{(p^2+4)^2)}$ } (6)

Q4. (a) Evaluate by Laplace transform

$\int_0^\infty t^3 e^{-t} \sin t \, dt = 0$

(8)

(b) State And Prove First and second shifting Theorem of Laplace Transform.

(8)

Section-C

Q5. (a) Let $\overline{f_n}(s)$ be Hankel transform of order-n of function f(t) and $\overline{f'_n}(s)$ is the Hankel transform of $f'(t)$. Then prove that $\overline{f'_n}(s) = \frac{-s}{2n}\left[(n+1)\overline{f_{n-1}}(s) - (n-1)\overline{f_{n+1}}(s)\right]$.

(10)

(b) State and prove Parseval's Theorem of Hankel transforms.

(6)

Q6. (a) Find the finite Hankel transform of x^2 if $x\,J_0$ (sx) is the kernel of the transform.

(6)

(b) The magnetic potential V for a circular disc of radius a and strength w , magnetized parallel to its axis , satisfies laplace equations is equal to $2\pi w$ on the disc itself and vanishes at exterior points in the plane of the disc , Show that at the point (r, z) ; z > 0

$$V = 2\pi w a \int_0^\infty e^{-sz}\, J_0(rs)\, J_1(as)\, ds$$

(10)

Section-D

Q7. (a) Using the Z- transform , Solve

$$u_{n+2} + 4\,u_{n+1} + 3\,u_n = 3^n \quad \text{with} \quad u_0 = 0\,,\, u_1 = 1.$$

(8)

(b) Find the inverse z-transform of $\frac{10z}{z^2-3z+2}$. (8)

Q8. (a) Use convolution theorem to evaluate $z^{-1}\left\{\frac{z^2}{(z+a)(z+b)}\right\}$.

(8)

(b) State and prove Shifting property of z-transform. (8)

Exam Code: 211003 **Paper Code: 3226**
(20)

Programme: Master of Science (Mathematics) Semester-III

Course Title: Statistics-I

Course Code: MMSL-3334 (Opt-III)

Time Allowed: 3 Hours **Max Marks: 80**

Note: Attempt five questions in all, selecting at least one question from each section, Fifth question can be attempted from any section. Each question carries equal marks.
Students can use only Non-Programmable& Non-Storage Type Calculator and statistical tables.

Section A

Q1. (a) What is Skewness . How does it differ from kurtosis? Also show that Bowley's coefficient of skewness always lies between -1 and 1. [8]

(b) Calculate missing frequency from the following data given that the median is 46.75 [8]

Marks	20-30	30-40	40-50	50-60	60-70
Students	3	5	20	?	5

Q2. (a) In a class with 65 boys and 40 girls, 16 boys and 24 girls have a hobby of dancing. A student is picked up at a random, find the probability

(i) That the student has dancing as a hobby.
(ii) That the student picked up is a dancer given that he is a boy. [8]

(b) Let X have probability mass functions as $f(x) = \frac{x}{15},\ x = 1,2,3,4,5;\ f(x) = 0, otherwise$. Find the distribution function of X. [8]

Section B

Q.3 The joint density function of two continuous random variables X and Y is given by

$$f(x,y) = \begin{cases} k(6-x-y); & 0 \le x < 2,\ \ 2 \le y < 4 \\ 0; & elsewhere \end{cases}$$

Find (i) k (ii) $P(X < 1 \cap Y < 3)$ (iii) $P(X + Y < 3)$ (iv) $P\{X < 1 | \ Y < 3\}$. [16]

Q4. The joint density function of two random variables X and Y is given by

$$f(x,y) = \begin{cases} \frac{2x+y}{210}; & 2 < x < 6,\ \ 0 < y < 5 \\ 0; & elsewhere \end{cases}$$

Find $(i) E(X)$ (ii) $E(Y)$ (iii) $E(XY)$ (iv) $E(X^2)$

$(v) E(Y^2)$ (vi) $V(X)$ $(vii) V(Y)$ $(viii) Cov(X,Y)$. [16]

Section C

Q5. (a) State and prove De Moivre's Laplace Theorem. [8]

(b) The density of random variable X is $f(x) = 1,\ 0 \le x \le 1$ and zero otherwise. Using Chebychev's inequality find the upper bound to $P[|X - 0.5| > 0.25]$ and compare it with exact probability. [8]

Q6. (a) Find mean and variance of Poisson distribution with parameter λ. [8]

(b) Using moment generating function find mean and variance of Geometric distribution. [8]

Section D

Q7. (a) Given a standard normal variate Z with $P(0 \le z \le 1) = 0.3413$, if X is a normal variate with mean 30 and standard deviation 5, find $P(|X - 30| > 5)$. [8]

(b) What do you understand by independence of attributes? Give a criterion for independence for attribute A and B. [8]

Q8 (a) Calculate the rank correlation coefficient for the following given ranks: [8]

Rank A	3	5	8	4	7	10	2	1	6	9
Rank B	6	4	9	8	1	2	3	10	5	7

(b) The lines of regression are given by $X + 2Y = 5,\ 2X + 3Y = 8$ and Var (Y) = 4. Calculate the values of $\bar{X},\ \bar{Y}, correlation\ coefficient\ r \quad and\ \ Var(X)$. [8]

Exam Code: 211003 **Paper Code: 3227**
(20)

Programme: Master of Science (Mathematics) Semester-III

Course Title: Operations Research-I

Course Code: MMSL-3335 (Opt-IV)

Time Allowed: 3 Hours **Max Marks: 80**

Note: Attempt five questions in all, selecting at least one question from each section. Fifth question may be attempted from any Section. Each question carries sixteen marks. Students can use Non-Programmable and Non-Storage type Calculator

Section-A

1(a) Show that if a LPP has a feasible solution then it also has a basic feasible solution.
(b) Prove that any convex combination of k different optimum solutions to LPP is again optimum solution to the problem. (8,8)

2. Use two-phase simplexmethod to

$$\text{Maximize } Z=5x_1-4x_2+3x_3$$
subject to the Constraints
$$2x_1+x_2-6x_3=20$$
$$6x_1+5x_2+10x_3 \leq 76,$$
$$8x_1-3x_2+6x_3 \leq 50,$$
$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$$

Is the solution unique? If not give two different solutions. (16)

Section-B

3(a) Prove that Dual of the Dual is Primal.
(b) Use Duality to solve the LPP:

$$\text{Minimize } Z = 60x_1 + 80x_2 \quad \text{subject to the constraints}$$
$$x_2 \geq 200$$
$$x_1 \leq 400$$
$$x_1+x_2=500 \;, x_1, x_2 \geq 0$$ (8,8)

4. Find the optimum integer solution to the following LPP using Gomory's Cutting Plane Method:

$$\text{Maximize } Z = x_1+4x_2$$
subject to the constraints
$$2x_1+4x_2 \leq 7$$
$$5x_1+3x_2 \leq 15 \;, x_1, x_2 \geq 0 \text{ and are integers}$$ (16)

Section-C

5. Find the starting solution of the following transportation problem by Vogel's Approximation method. Also obtain an optimum solution. (Table 1.1)

	D	E	F	G	Supply
A	3	7	6	4	5
B	2	4	3	2	2
C	4	3	8	5	3
Demand	3	3	2	2	

(Table 1.1)

(16)

6(a) A company is faced with the problem of assigning 4 machines to 6 different jobs (one machine to one job only). The profits are estimated as follows: (Table 1.2). Solve the problem to maximize the total profits.

Job	Machine			
	A	B	C	D
1	3	6	2	6
2	7	1	4	4
3	3	8	5	8
4	6	4	3	7
5	5	2	4	3
6	5	7	6	4

(Table 1.2)

(b) Prove that the number of basic variables of the general transportation problem at any stage of feasible solution must be (m+n-1). (8,8)

Section-D

7(a) Use the principle of dominance to solve the following 5× 5 game:

Player B

Player A

8	12	14	10	11
9	11	15	10	13
7	8	6	11	12
10	9	7	9	9
12	13	10	12	10

(b) Solve the following LPP using Dynamic Programming approach:

Maximize $Z = 2x_1 + 5x_2$

subject to the constraints

$2x_1 + x_2 \leq 43$

$2x_2 \leq 46$, $x_1, x_2 \geq 0$ (8,8)

8. Divide a positive quantity 'c' into 'n' parts in such a way that their product is maximum. (use Dynamic Programming) (16)