Exam Code: 211004 **Paper Code: 4217**
(20)

Programme: Master of Science (Mathematics)
Semester-IV

Course Title: Functional Analysis-II

Course Code: MMSL-4331

Time Allowed: 3 Hours **Max Marks: 80**

Note: Attempt five questions in all, selecting at least one question from each section. Fifth question can be attempted from any section. Each question carries equal marks.

Section -A

1. Prove that if $(e_1, e_2, \ldots\ldots)$ is an orthonormal sequence in a Hilbert space, then $e_n \to 0$ weakly. Also prove that if $< x_n >$ converges weakly to x and $\| x_n \| \to \|x\|$ then $< x_n >$ converges strongly to x. (16)

2. (a)Prove that the adjoint T* is linear and bounded. Also Prove $\|T^*\|=\|T\|$ and $\|T^*T\|=\|T\|^2$
(b) An operator T on a Hilbert space H is a self adjoint iff $<Tx,x>$ is real. (8,8)

Section -B

3(a) Prove that the resolvent set $\rho(T)$ of a bounded linear operator T on a complex Banach space X is open.
(b) Let A ∈ B(H) be a self-adjoint operator, then every eigen value of A are real, where H is a Hilbert space. (8,8)

4(a) Prove that the spectrum $\sigma(T)$ of a bounded linear operator T: X→ X on a complex Banach space is Compact.
(b) Let T ∈ B(H) be a normal operator. Then if $\lambda \neq \mu$, the null spaces N(T-λ) and N (T-μ) are orthogonal to one another. (8,8)

Section- C

5. Let T: X→ Y be a linear operator where X and Y are normed spaces. If T is compact, then its adjoint operator $T^*: Y^* \to X^*$ is also compact where X^* and Y^* are dual space of X and Y respectively. (16)

6. Define compact linear operator. Let X and Y be normed spaces, then prove that
(i) Every compact linear operator T: X→ Y is bounded and hence continuous.
(ii) If dim X=∞, then the identity operator I: X→ X is not compact. (8,8)

Section -D

7(a) Prove that $\sigma(x) \neq \emptyset$ for any $x \in A$ where A is a complex Banach algebra with identity e. (8,8)
(b) Prove that no topological divisor of zero is invertible.

8. Let A be a complex Banach algebra with identity e. then for any $x \in A$, Prove that the spectrum $\sigma(x)$ is compact and the spectral radius satisfies $\gamma_\sigma(x) \leq ||x||$ (16)

Exam Code: 211004 **Paper Code: 4218**
(20)

Programme: Master of Science (Mathematics)
Semester-IV

Course Title: Topology-II

Course Code: MMSL-4332

Time Allowed: 3 Hours **Max Marks: 80**

Note: Attempt five questions in all, selecting at least one question from each section. The fifth question may be attempted from any section. Each question carries equal marks.

Section- A

1. (a) Define Completely Regular Topological space. Also prove a normal regular space is completely regular. (8)
   (b) Prove that every metric space is completely regular space. (8)

2. Define Completely normal topological space with the help of an example. Prove that every metric space is completely normal and hence T_5. (16)

Section- B

3. (a) If every cover of X by base elements has a finite subcover, then prove that X is compact. (8)
   (b) Prove that every compact Hausdorff space is Normal space and hence T_4 space. (8)

4. (a) Let $(X_\alpha, \tau_\alpha)_{\alpha \in \Lambda}$ be topological spaces and (X, τ) be their product space then (X, τ) is compact iff each (X_α, τ_α) is compact. (8)

   (b) Define Countably compact space. Prove that a topological space is countably compact iff each sequence in X has a cluster point. (8)

Section- C

5. State and prove Stone-Cech compactification theorem. (16)
6. (a) Prove that metrizablility is hereditary property. (8)
   (b) Prove that a topological space (X, τ) is Tychonoff iff it is homeomorphic to some subspace of Tychonoff cube. (8)

Section- D

7. (a) A point x in a topological space (X, τ) is a cluster point of a net S in X iff some subnet of S converges to x. (8)
   (b) A topological space (X, τ) is Hausdorff iff limit point of each net in it is unique. (8)

8. (a) State and prove characterization of compactness in terms of filters. (8)

   (b) Prove that every filter on X is contained in an ultrafilter on X. (8)

Exam Code: 211004 **Paper Code: 4219**
(20)

Programme: Master of Science (Mathematics)
Semester-IV

Course Title: Number Theory

Course Code: MMSL-4333 (Opt-VII)

Time Allowed: 3 Hours **Max Marks: 80**

Note: Attempt five questions in all selecting at least one question from each section. The fifth question may be attempted from any section. All question carry equal marks (16 marks each).

Section A

Q1. (a) Using Wilson's theorem solve $2(26)!\ (mod\ 29)$.

(b) State and prove Chinese Remainder Theorem to solve the system of given congruences.

[8x2=16]

Q2. (a) If n has primitive roots, then show that it has $\emptyset\ (\emptyset(n))$ primitive roots.

(b) Find all primitive roots modulo 5^2. [8x2=16]

Section B

Q3. (a)Solve $7x^3 \equiv 3\ (mod\ 11)$, using the index table, with 2 as the smallest primitive root of 11.

(b) State and Prove Gauss Quadratic Reciprocity Law for any set of natural numbers m and n.

[8x2=16]

Q4. (a) Is the congruence $x^2 \equiv 51\ (mod\ 71)$ solvable? Justify your answer.

(b) Evaluate $\left(\frac{137}{401}\right)$. [8x2=16]

Section C

Q5. (a) Verify whether $\sum_{d|m} \log d = \frac{1}{2}\ \tau(m)$ for all positive integers m.

(b) Find all solutions of $x^2 + y^2 = z^2$, such that $0 < z < 30$. [8x2=16]

Q6. (a) Prove that the Mobius function (μ) is multiplicative.

(b) Verify whether $\sum_{d|n} \mu(d)\tau(d) = (-1)^r$, where n has r distinct prime divisors and the symbols have the usual meaning. [8x2=16]

Section D

Q7 (a) Evaluate [1, 3, 1, 2, 1, 2,......].
(b) Find the simple continued fraction for $\sqrt{13}$ and solve Pell's equation $x^2 - 13y^2 = -1$.

[8x2=16]

Q8.(a) State and prove Hurwitz theorem for any irrational number ξ.

(b) Find the following and preceding terms of $\frac{4}{9}$ in F_{13}, the set of Farey fractions of order 13.

[8x2=16]

Exam Code: 211004 (20)

Paper Code: 4220

Programme: Master of Science (Mathematics) Semester-IV

Course Title: Statistics-II

Course Code: MMSL-4334 (Opt-VIII)

Time Allowed: 3 Hours

Max Marks: 80

Note: Attempt five questions in all, selecting one question from each section. Fifth question can be attempted from any section. Each question carries 16 marks. Students are allowed to use statistical Tables and a simple non programmable or non- storage calculator.

Section -A

Q-1 a)Let $y_1 < y_2 < y_3 < y_4$be the order statistics of a random sample of size 4 from a distribution having pdf f(x)=e^{-x},x> $0. Find\ P(y_4\ \leq 3)$

b) Prove that ratio of two independent Chi -Square Variate is a Beta Variate of second kind

8x2

Q-2 (a) Derive odd ordered and even ordered Moments of t-distribution

(b)Let X_1 , X_2 ,X_3 …….. X_n be a random sample from N(0,1)

Define $\bar{X}_K$ $=\sum_1^K X_i$ /K and $\bar{X}_{N-K}$ $=\sum_{K+1}^N X_i$ /N-K

Find the distribution of $\frac{X_1^2}{X_2^2}$ and $\frac{X_1}{X_2}$

8x2

Section -B

Q -3 (a) State and prove invariance property of consistent estimators

(b) If T is the most efficient estimator of θ and T_1 another estimator with efficency 'e ',then

Var(T-T_1) = $\frac{1-e}{e}$ $Var(T)$

8x2

Q-4 (a)Find the minimum Variance bound Estimator for μ in case of Poission Distribution

(b) Obtain the most powerful test of size α for testing Ho:μ=μo against H1: μ=$\mu 1$ where: $\mu 1$>μo,if the p.d.f of the random variable X is f(x, : $\mu)$=$\frac{1}{\sqrt{8\pi}}e^{\frac{-1(x-\mu)^2}{8}}$,-∞ <$x$<∞

Section C

Q- 5 (a) Two Sample of sizes 10 and 12 taken from two normal populations give S_1=12 and S_2=18. Test the Hypothesis thatσ_1= σ_2

b) In an Experiment on pea -breeding obtained the following frequencies of seeds;315 round and yellow,101 wrinkled and yellow ,108round and green and 32 wrinkled and green. According to his theory of heredity should be in the ratio 9:3:3:1. Is there any evidence to doubt his theory at 5 % level of significance.

8x2

Q- -6 (a) Explain, Stating clearly the assumptions involved, the t-test for testing the significance of the difference between two sample means.

(b) Find likelihood ratio test for the Variance of a normal distribution 8x2

Section D

Q-7 A study was conducted and number of days of recovering a fractured bone were noted for 15 patients having different physical fitness

Physical fitness	Number of days of recovery
Poor	60 55 52 53
Average	55 45 49 46 48 50
Good	40 44 46 41 50

Test the claim that prior physical fitness has an influence on the time period of recovering from a fractured bone at 5 % 16

Q-8 Develope a procedure for Analysis of variance in two way classified data with one observation per cell 16

Exam Code: 211004 Paper Code: 4221
(20)

Programme: Master of Science (Mathematics)
Semester-IV

Course Title: Operations Research-II

Course Code: MMSL-4335 (Opt-IX)

Time Allowed: 3 Hours **Max Marks: 80**

NOTE: Candidates are required to attempt five question by selecting at least one question from in each section. The fifth question may be attempted from any section. Each question carries 16 marks. The students can use only Non-Programmable & Non-Storage type calculator.

SECTION-A

1. (a) Discuss briefly the elements of queuing system.
(8 Marks)

(b) A T.V. repairman finds that the time spent on his jobs has an exponential distribution with mean 30 minutes. If he repairs sets in the order in which they came in, and if the arrival of sets is approximately Poisson with an average rate of 10 per 8-hour day. What is repairman's expected idle time each day? How many jobs are ahead of the average set just brought in?

(8 Marks)

2. (a) At a railway station, only one train is handled at a time. The railway yard is sufficient only for two trains to wait while other is given signal to leave the station. Trains arrive at the station at an average rate of 6 per hour and the railway station can handle them on an average of 12 per hour. Assuming Poisson arrivals and exponential service distribution, find the steady-state probabilities for the various number of trains in the system. Also find the average waiting time of a new train coming into the yard. (8 Marks)

(b) Explain the generalized model: Birth-Death process.

(8 Marks)

SECTION-B

3. (a) Discuss various factors affecting inventory control.

(8 Marks)

(b) Neon lights in an industrial park are replaced at the rate of l00 units per day. The physical plant orders the neon lights periodically. It costs Rs. 100 to initiate a purchase order. A neon light kept in storage is estimated to cost about Re. 02 per day. The lead time between placing and receiving an order is 12 days. Determine the optimum inventory policy for ordering the neon lights. (8 Marks)

4. (a) Explain the concept of deterministic inventory problem with shortages. (8 Marks)

(b) Find the optimum order quantity for a product for which the price breaks are as follows:

Quantity	Unit Cost (Rs.)
$0 \leq Q_1 < 500$	10.00
$500 \leq Q_2$	9.25

The monthly demand for the product is 200 units, the cost of storage is 2% of the unit cost and the cost of ordering is Rs. 350.00. (8 Marks)

SECTION-C

5. (a) Machine A costs Rs. 9000. Annual operating costs are Rs. 200 for the first year, and then increase by Rs. 2000 every year. Determine the best age at which to replace the machine. If the optimum replacement policy is followed, what will be the average yearly cost of owning and operating the machine? (8 Marks)

(b) Discuss the replacement policy of an equipment that fails suddenly. (8 Marks)

6. (a) Calculate the probability of a staff resignation in each year from the following table:

Year	0	1	2	3	4	5	6	7	8	9	10
Staff at year end	1000	940	820	580	400	280	190	130	70	30	0

(8 Marks)

(b) Explain the concept of equipment renewal problem.

(8 Marks)

SECTION-D

7. (a) Discuss the following:

(i) Need of simulation

(ii) Methodology of simulation. (8 Marks)

(b) Customers arrive at a milk booth for the required service. Assume that inter-arrival and service times are constant and given by 1.8 and 4 time units respectively. Simulate the system by hand computations for 14 time units. (8 Marks)

8. (a) Write a short note on Monte-Carlo simulation.

(8 Marks)

(b) Bright bakery keeps stock of a popular brand of cake. Previous experience indicates the daily demand as given here:

Daily demand	0	10	20	30	40	50
Probability	.01	.20	.15	.50	.12	.02

Consider the following sequence of random numbers:

48, 78, 19, 51, 56, 77, 15, 14, 68, 09

Using the sequence, simulate the demand for the next 10 days. (8 Marks)